AQA A-Level Further Mathematics Warmup - Paper 1 2022

What are the definitions of the hyperbolic functions $\sinh(x)$, $\cosh(x)$ and $\tanh(x)$?	Show that $\lambda = -1$ and $\mathbf{v} = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$ are an eigenvalue-eigenvector pair for $\mathbf{A} = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 1 & 4 \\ 4 & 1 & 1 \end{pmatrix}$	Given that $1 + 3\mathrm{i}$ is a root of $p(z) = z^3 - 5z^2 + 16z - 30$, fully factorise $p(z)$.	Plot the complex loci satisfied by $\lvert z - 4 + 2i \rvert = 3$	How does the discriminant of the auxiliary equation for damped harmonic motion determine the type of damping?
State the modulus and argument form, and the exponential form of the complex number $z = a + \mathrm{i}b$.	Prove that $\operatorname{arcosh}(x) = \ln\left(x + \sqrt{x^2 - 1}\right)$	State the scalar product of two vectors $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$	How do you find the angle between a plane and a line?	Find the inverse of $\mathbf{A} = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 1 & 4 \\ 4 & 1 & 1 \end{pmatrix}$
What is the formula for finding the area enclosed by a polar curve?	Solve $z^3 = 1$	For the 2nd order ODE $a\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + b\dfrac{\mathrm{d}y}{\mathrm{d}x} + cy = 0$ describe how the discriminant of the auxiliary equation $am^2 + bm + c = 0$ determines the general solution.	Plot $y = \tanh(x)$	Sketch $r = 2 - 2\cos(\theta)$
In the context of hyperbolic functions describe Osborn's rule and the affect this has on the identity $\cos^2(x) + \sin^2(x) = 1$	What are the key properties of Simple Harmonic Motion (SHM)?	Find the characteristic polynomial of the matrix $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$	What is derivative of $y = \cosh(x^2 + 1)$?	How can you convert a polar coordinate (r, θ) into cartesian coordinates?

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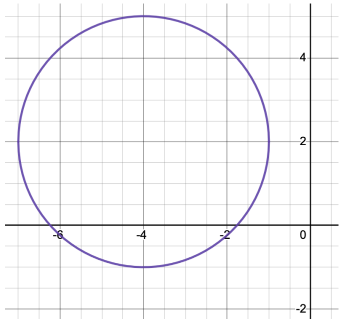
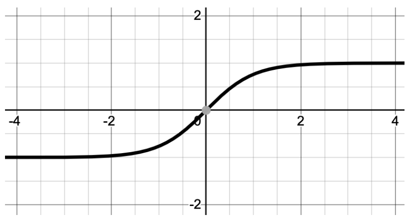
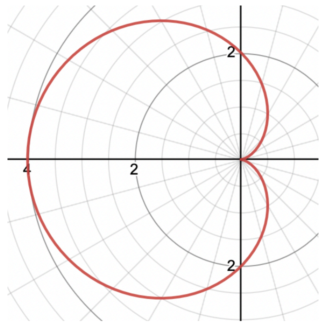
$\sinh(x) = \frac{e^x - e^{-x}}{2}$ $\cosh(x) = \frac{e^x + e^{-x}}{2}$ $\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$	Show that $\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$	$p(z) = (z-3)(z^2 - 2z + 10)$		Discriminant less than zero means the system is lightly damped. Discriminant equal to zero means the system is critically damped. Discriminant greater than zero is heavily damped.
$z = re^{i\theta}$ and $z = r(\cos(\theta) + i\sin(\theta))$ where r is the modulus of z and θ is its argument.	Proof.	$\mathbf{a}\cdot\mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos(\theta)$	Find the angle between the the normal to the plane and the line and then do 90° minus this angle.	$\mathbf{A}^{-1} = \begin{pmatrix} -3 & 1 & 2 \\ 14 & -5 & -8 \\ -2 & 1 & 1 \end{pmatrix}$
$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 \, d\theta$	$z = 1$ $z = \frac{-1 + i\sqrt{3}}{2}$ $z = \frac{-1 - i\sqrt{3}}{2}$	$b^2 - 4ac > 0$, distinct real roots α, β so $y = Ae^{\alpha x} + Be^{\beta x}$. $b^2 - 4ac = 0$, repeated real root α so $y = (A + Bx)e^{\alpha x}$. $b^2 - 4ac < 0$, complex roots $p \pm qi$ so $y = e^{px}(A\cos(qx) + B\sin(qx))$		
For every product or implied product of sines the sign is changed. $\cosh^2(x) - \sinh^2(x) = 1$	$\ddot{x} = -\omega^2 x$. Period $\frac{2\pi}{\omega}$. The force acting on the object undergoing SHM is proportional to its displacement but in the opposite direction. The general solution $x = A\cos(\omega t) + B\sin(\omega t)$ can be rewritten in the form $x = R\cos(\omega t - \phi)$.	$\lambda^2 - 6\lambda + 5$	$\frac{dy}{dx} = 2x\sinh(x^2 + 1)$	Use $x^2 + y^2 = r^2$, $x = r\cos(\theta)$, $y = r\sin(\theta)$ and $\tan(\theta) = \frac{y}{x}$