AQA A-Level Further Maths 2022 Paper 1

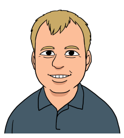
Do nut turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: / 100



1 Let $z = 3\left(\cos\left(\frac{\pi}{4}\right) + \mathrm{i}\sin\left(\frac{\pi}{4}\right)\right)$ and $w = 4\left(\cos\left(\frac{7\pi}{6}\right) + \mathrm{i}\sin\left(\frac{7\pi}{6}\right)\right)$ then zw equals

$$12\left(\cos\left(\frac{17\pi}{12}\right) + \mathrm{i}\sin\left(\frac{17\pi}{12}\right)\right)$$

$$7\left(\cos\left(\frac{17\pi}{12}\right) + \mathrm{i}\sin\left(\frac{17\pi}{12}\right)\right)$$

$$12\left(\cos\left(\frac{-11\pi}{12}\right) + \mathrm{i}\sin\left(\frac{-11\pi}{12}\right)\right)$$

$$7\left(\cos\left(\frac{-11\pi}{12}\right) + \mathrm{i}\sin\left(\frac{-11\pi}{12}\right)\right)$$

[1 mark]

2 The derivative, with respect to x of $\cosh(4x)$ is

$\sinh(4x)$ $\quad$ $4\sinh(4x)$ $\quad$ $4\cosh(4x)$ $\quad$ $2\sinh(4x)$

[1 mark]

3 A particle performing simple harmonic motion has a speed of 5 ms-1 when it is 2 m away from the centre of oscillation. If the amplitude is 4 m, what is the period of oscillation?

$$\frac{12\pi}{5\sqrt{3}} \qquad \frac{10\sqrt{3}}{6} \qquad \frac{12}{5\sqrt{3}} \qquad \frac{10\pi}{\sqrt{3}}$$

[1 mark]

4 **a)** Prove that the logarithmic form of $y = \text{arsinh}(x)$ is $y = \ln\left(x + \sqrt{x^2 + 1}\right)$

[4 marks]

b) Solve $3\cosh^2(x) - 7\sinh(x) - 9 = 0$, giving your answers in exact logarithmic form.

[4 mark]

5 **a)** Show that $z = 2 + \mathrm{i}$ is a root of the polynomial
$p(z) = z^4 - 4z^3 + 14z^2 - 36z + 45.$

[2 marks]

b) State another complex root of $p(z)$.

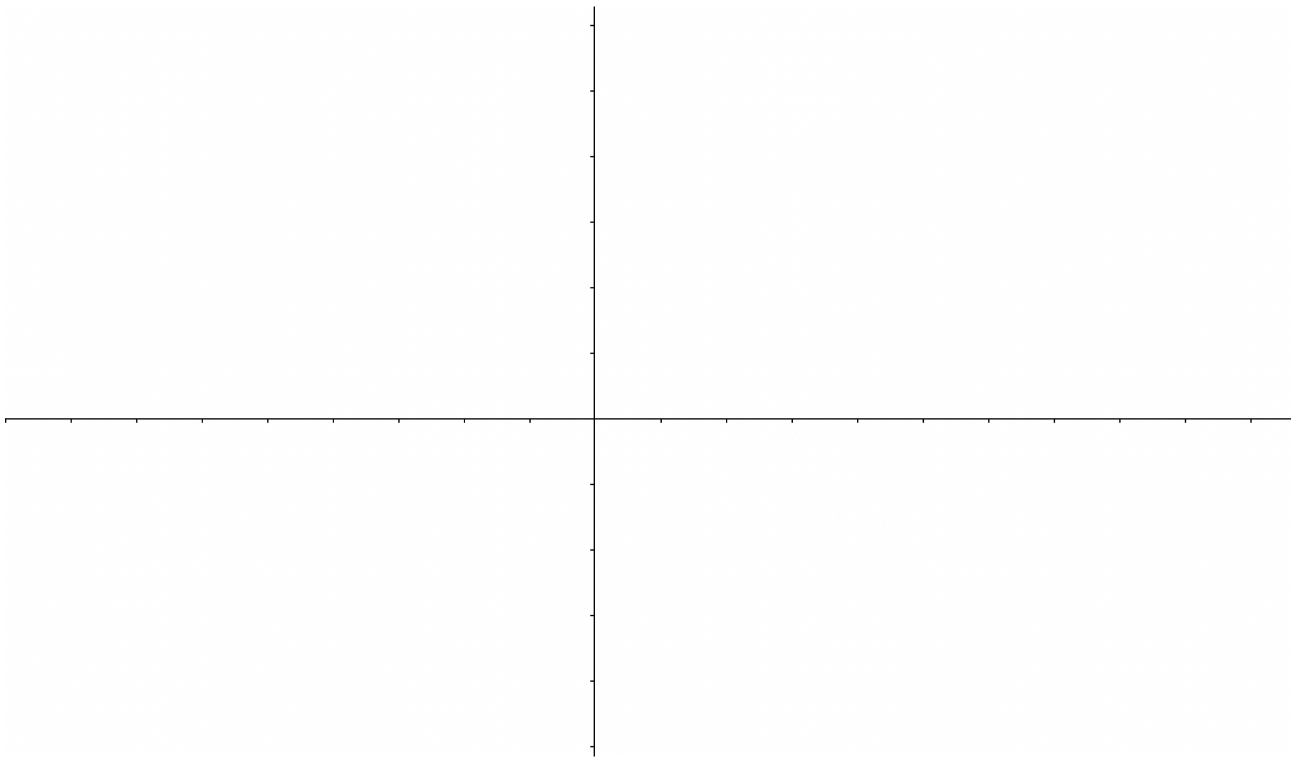
[1 mark]

c) Hence, show that the two remaining roots of $p(z)$ are purely imaginary.

[4 marks]

d) Plot the four roots on the Argand diagram below.

[2 marks]



6 Consider the matrix $T = \begin{pmatrix} 5 & 3 \\ 2 & 4 \end{pmatrix}$

a) Find $|T|$ and explain its geometrical significance

[2 marks]

b) Find the eigenvalues, and associated eigenvectors of the 2×2 matrix T.

[5 marks]

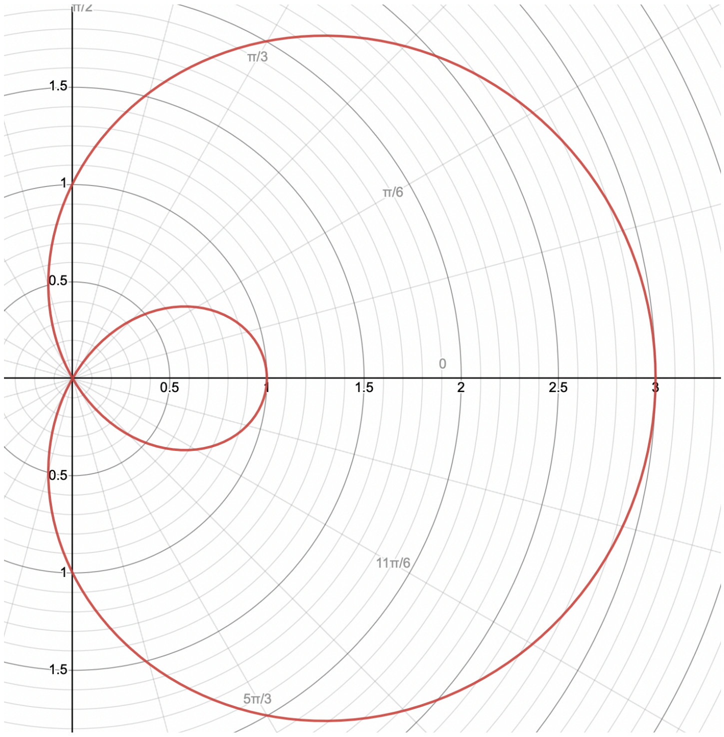
c) Describe the relationship between the eigenvectors of a matrix and the invariant lines of the same matrix.

[1 mark]

d) Find, with full justification, the invariant lines of $T = \begin{pmatrix} 5 & 3 \\ 2 & 4 \end{pmatrix}$

[4 marks]

7 **a)** Sketch the curve $r = 1 + 2\cos(\theta)$



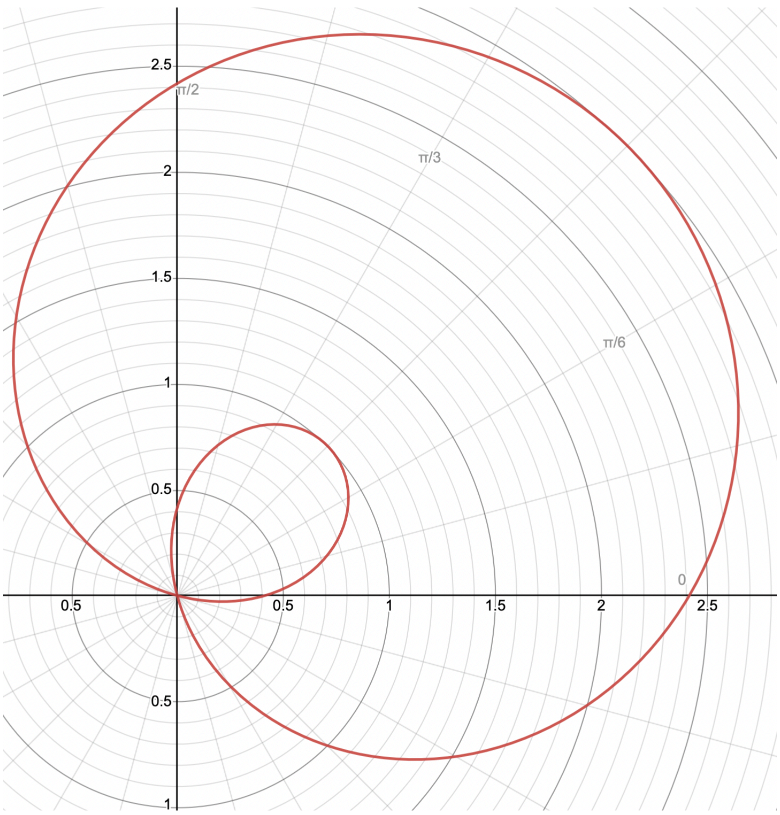
[2 marks]

b) Find the total area contained between the outer and inner loops of the curve sketched in (a), giving your answer as an exact value.

[7 marks]

c) Sketch the curve $r = 1 + 2\cos\left(\theta - \frac{\pi}{4}\right)$

[2 marks]



8 **a)** Find the angle between the line, l, with equation

$\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$ and the plane with equation

$\mathbf{r} \cdot (2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = 4$

[5 marks]

b) Find the shortest distance from the point $P(2,2,4)$ to the plane with equation $2x + 3y - z - 4 = 0$

[4 marks]

9 Find the possible angles between consecutive roots of the equation

$$1 + z + z^2 + z^3 + z^4 + z^5 = 0$$

[10 marks]

10 A complex number, z, is represented by the point P in the complex plane.

a) Given that $|z - 3 - 4\mathrm{i}| = 4$, sketch the locus of P.

[2 marks]

b) Write down the Cartesian equation of this locus.

[1 mark]

c) Find, justifying your reasoning, the minimum value of $|z|$ and the maximum value of $|z|$.

[4 marks]

d) Write down, in Cartesian form, the coordinates of the points where these minimum and maximum values of $|z|$ occur.

11 **a)** Find the inverse of the matrix $\mathbf{A} = \begin{pmatrix} 1 & 2 & a \\ -2 & 3 & 1 \\ 1 & 1 & 2 \end{pmatrix}$

[5 marks]

b) Consider the following three planes:

$$\Pi_1 : x + 2y + 3z = 2$$
$$\Pi_2 : -2x + 3y + z = 5$$
$$\Pi_3 : x + y + 2z = 4$$

i) Explain why, using part (a), that these planes do not intersect at a unique point.

[2 marks]

ii) What is the geometric configuration of these three planes?

[4 marks]

12 A particle, P, of mass m, is suspended from a fixed point O by a light elastic string of natural length 2 m and modulus of elasticity $40m$.

When the particle hangs in equilibrium vertically below O, the extension of the string is $\dfrac{g}{20}$ m.

At time $t = 0$ s, the particle is projected vertically downwards from the equilibrium position with speed $2U$.

During its subsequent motion, when P is moving with speed v it experiences a resistive force of $4mv$.

The displacement of P below its equilibrium position at time t is x.

a) Show that while the string is taut x satisfies the equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 20x = 0$$

[5 marks]

b) Find x in terms of U and t.

[7 marks]

c) What kind of damping is being caused by this resistive force?

[1 mark]

d) Find the speed, in terms of U and π, of P when it first returns to its equilibrium position.

[4 marks]