AQA A-Level Maths 2022 Paper 3

Do not turn over the page until instructed to do so.

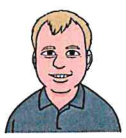
This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: / 100

Solutions



1 Given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{8x^{\frac{3}{2}}}$ find an expression for y

$y = -\dfrac{1}{4\sqrt{x}} + C$

$y = \dfrac{1}{4\sqrt{x}} + C$

$y = \dfrac{1}{12x^{\frac{5}{2}}} + C$

$y = -\dfrac{3}{16x^{\frac{5}{2}}} + C$

[1 mark]

2 For what values of $|x|$ is the binomial expansion of $(2 + 5x)^{\frac{1}{3}}$ valid.

$|x| < \dfrac{2}{5}$

$|x| < 1$

$|x| < 2$

$|x| < \dfrac{5}{2}$

[1 mark]

3 The area between the curves $y = x^3$ and $y = x^{\frac{1}{3}}$ is M. What is the area between $y = x^3 + 2$ and $y = x^{\frac{1}{3}} + 2$?

$-M$ $M + 2$ M $M - 2$

[1 mark]

4 **a)** Let $f(x) = \dfrac{3x+2}{x+3}$. Find $f^{-1}(x)$.

[3 marks]

Let $y = \dfrac{3x+2}{x+3}$

$\Rightarrow xy + 3y = 3x + 2$

$\Rightarrow x(y-3) = 2 - 3y$

$\Rightarrow x = \dfrac{2-3y}{y-3}$

Hence, $f^{-1}(x) = \dfrac{2-3x}{x-3}$, $x \neq 3$

b) Why does the function $g(x) = x^2 + 4x + 8, \quad -10 \le x \le 10$ not have an inverse.

[1 mark]

g(x) is a many to one function in the range $-10 \le x \le 10$ and so cannot have an inverse

5 Prove, by contradiction, the arithmetic-geometric mean inequality,

$$\frac{1}{2}(a+b) \ge \sqrt{ab}, \quad a, b \ge 0,\ a \ne b$$

[5 marks]

We wish to prove $\frac{1}{2}(a+b) \geq \sqrt{ab}$ ①.

Assume, for a contradiction, that ① is false, and so

$$\frac{1}{2}(a+b) < \sqrt{ab}$$

$$\Rightarrow \frac{1}{4}(a+b)^2 < ab$$

$$\Rightarrow (a+b)^2 < 4ab$$

$$\Rightarrow a^2 + 2ab + b^2 < 4ab$$

$$\Rightarrow a^2 - 2ab + b^2 < 0$$

$$\Rightarrow (a-b)^2 < 0$$

which is a contradiction, since anything ~~sqared~~ squared must be greater than, or equal to, 0.
As our only assumption was that ① was false we must conclude that in fact ① is true and we have proved the result

6 Plastic balls are manufactured in such a way that the volume, V increases at a constant rate of b cm³ per second.

a) Find the rate of change of the surface area, A, of the ball in terms of r and b.

[4 marks]

$$\frac{dV}{dt} = b$$

Surface area of sphere $= 4\pi r^2$

Volume of sphere $= \frac{4}{3}\pi r^3$

By the chain rule,

$$\frac{dV}{dt} = \frac{dV}{dA}\frac{dA}{dt} \quad (*)$$

Let $A = 4\pi r^2 \Rightarrow r = \sqrt{\frac{A}{4\pi}}$, then

$$V = \frac{4}{3}\pi\left(\frac{A}{4\pi}\right)^{3/2} = \frac{4}{3}\times\frac{1}{8}\frac{\pi A^{3/2}}{\pi^{3/2}}$$

$$= \frac{1}{6\sqrt{\pi}} A^{3/2}$$

so $\frac{dV}{dA} = \frac{3}{2}\times\frac{1}{6\sqrt{\pi}} A^{1/2} = \frac{1}{4\sqrt{\pi}} A^{1/2}$

Now, using $(*)$

$$\frac{dV}{dt} = \frac{dV}{dA}\frac{dA}{dt} \Rightarrow \frac{dV}{dt} = \frac{A^{1/2}}{4\sqrt{\pi}}\frac{dA}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \frac{4\sqrt{\pi}}{\sqrt{A}}\frac{dV}{dt}$$

$$= 4b\sqrt{\frac{\pi}{A}}$$

b) If the rate of change of area is 2 cm^2 per second when the area is 4π cm^2, find the numerical value of the rate of change of the volume at this instant.

[2 marks]

$$2 = 4b\sqrt{\frac{\pi}{4\pi}}$$

$$\Rightarrow 2 = 4\sqrt{\frac{1}{4}}\, b$$

$\Rightarrow b = 1$ cm^3 per second

7 The path of a rocket at time, t, seconds is modelled by the parametric equations

$$x = 10(2t)^{\frac{1}{2}}$$
$$y = 40t - 4t^2$$

a) What feature of these equations tells you that the rocket is not a projectile which is only acted upon by the force of gravity?

[1 mark]

The $(2t)^{1/2}$ means that the rocket does not follow a parabolic path.

b) Find the maximum height attained by the rocket.

[4 marks]

$x = 10(2t)^{1/2}$

$\frac{dx}{dt} = \frac{1}{2} \times 10(2t)^{-1/2} \times 2$

$= \frac{10}{\sqrt{2t}}$

$y = 40t - 4t^2$

$\frac{dy}{dt} = 40 - 8t$

Hence, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$= \frac{40 - 8t}{\frac{10}{\sqrt{2t}}}$ $= 40\sqrt{2t} - 8t\sqrt{2t}$

At stationary points $\frac{dy}{dt} = 0$, so

$40\sqrt{2t} - 8t\sqrt{2t} = 0$

$\Rightarrow 8\sqrt{2t}(5-t) = 0$

$\Rightarrow t = 0$ or $t = 5$

When $t = 5$, $x = 10\sqrt{10}$ and $y = 100$.

Hence, ~~the~~ maximum height attained by the projectile is 100m

c) Explain, with justification why this model is not valid for $t \geq 10$.

[1 mark]

For $t \geq 10$, $y < 0$ and so the rocket has hit the ground by then

8 **a)** A curve has equation $y = a\sin(x) + b\cos(x)$ where a and b are both positive constants.

The maximum value of y is 3 and the curve passes through the point $\left(\frac{\pi}{2}, \frac{3\sqrt{3}}{2}\right)$.

Find the exact values of a and b.

[4 marks]

Consider

$$R\sin(x+\alpha) = a\sin(x) + b\cos(x)$$

$$\Rightarrow R\sin(x)\cos(\alpha) + R\cos(x)\sin(\alpha) = a\sin(x) + b\cos(x)$$

$\sin(x)$ | $R\cos(\alpha) = a$

$\cos(x)$ | $R\sin(\alpha) = b$

But maximum value is $3 \Rightarrow R = 3$, and it passes through $\left(\frac{\pi}{2}, \frac{3\sqrt{3}}{2}\right) \Rightarrow \alpha = \frac{\pi}{6}$, hence

$$y = 3\sin\left(x + \frac{\pi}{6}\right)$$

and

$$a = 3\cos\left(\frac{\pi}{6}\right) = \frac{3\sqrt{3}}{2}$$

$$b = 3\sin\left(\frac{\pi}{6}\right) = \frac{3}{2}$$

So

$$y = \frac{3\sqrt{3}}{2}\sin(x) + \frac{3}{2}\cos(x)$$

b) Hence, find the solutions to $3\sqrt{3}\,\sin(x) + 3\cos(x) = 0$ for $0 \le x \le 2\pi$.

[2 marks]

$$3\sqrt{3}\sin(x) + 3\cos(x) = 2\left(\frac{3\sqrt{3}}{2}\sin(x) + \frac{3}{2}\cos(x)\right)$$

hence solutions are solutions ~~to~~

$$3\sin\left(x + \frac{\pi}{6}\right) = 0$$

$$\Rightarrow x = \frac{5\pi}{6}, \frac{11\pi}{6}$$

9 Geoff bought a classic car in 1996 for £9800.

A classic car specialist valued the car at 5 year intervals as shown in the table below.

Year	1996	2001	2006	2011
Value	9800	19500	39100	79000

The valuer suggests that the valuation price can be modelled by the equation $V = a \times b^t$, where t is the number of years after 1996.

a) Find the linearised form of the model given above.

[2 marks]

$$\log_{10}(V) = \log_{10}(a \times b^t)$$

$$= \log_{10}(a) + \log_{10}(b^t)$$

$$= \log_{10}(a) + t\log_{10}(b)$$

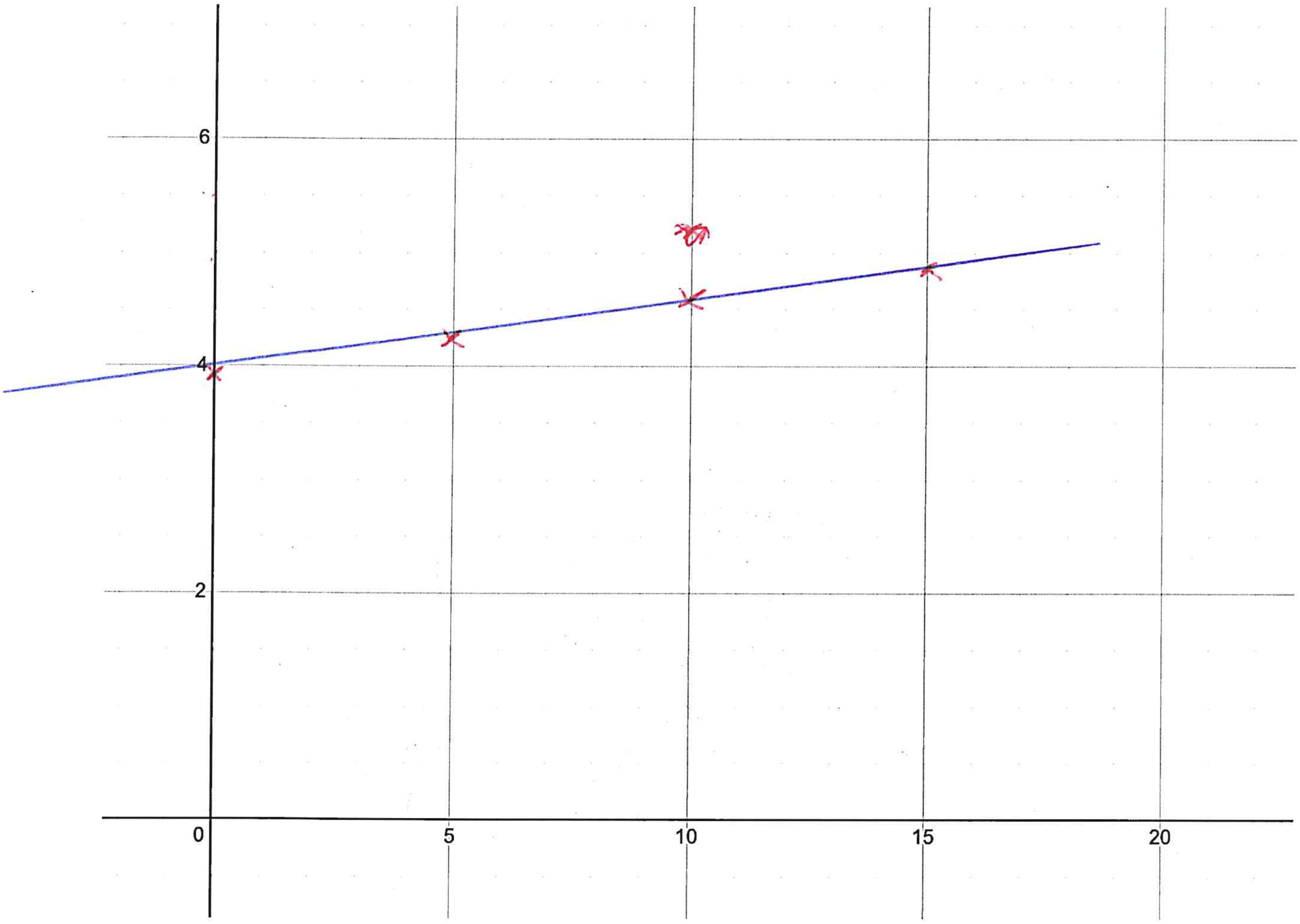
$$= \log_{10}(b)t + \log_{10}(a)$$

b) Complete the table below:

t	0	5	10	15
$\log_{10}(V)$	3.99	4.29	4.59	4.90

c) By plotting a graph below, estimate the values of a and b.

[4 marks]



$\log_{10}(b) = 0.06 \Rightarrow b = 1.148153621$
≈ 1.14

$\log_{10}(a) = 4.0 \Rightarrow a = 10000$

d) Find the expected value of the car in 2032 and comment on the validity of this value.

[2 marks]

$2032 \Rightarrow t = 36$

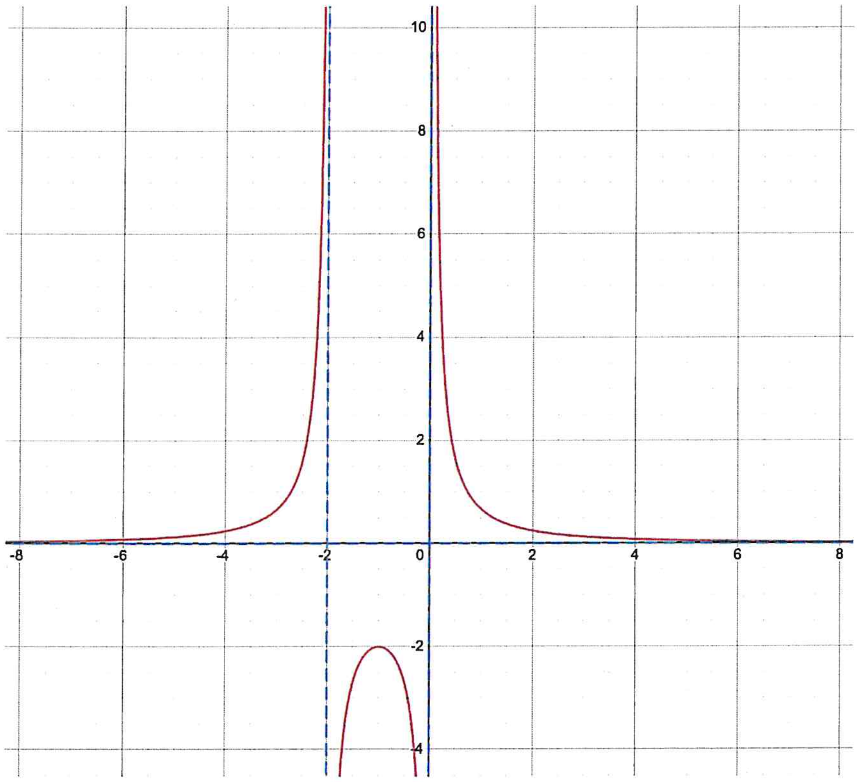
$V = 10000 \times 1.14^{36}$

$= 1118324.033$

$\approx £1100000$

$t = 32$ is significantly out of the range of the data collected. There is no guarantee that the valuation will continue to fit this model.

10 Consider $f(x) = \dfrac{2}{x^2 + 2x}$, the graph of which is shown below.



a) Find the coordinates of the stationary point.

[4 marks]

$$f'(x) = \frac{-4(x+1)}{x^2(x+2)^2}$$

Stationary points occur when $f'(x) = 0$,

so ~~x=~~

$$\frac{-4(x+1)}{x^2(x+2)^2} = 0$$

$\Rightarrow -4(x+1) = 0$

$\Rightarrow x = -1$

When $x = -1$, $y = -2$

So the stationary point is $(-1, -2)$

b) Hence, state the domain and range of the function, $f(x)$.

[2 marks]

Domain: $x \in \mathbb{R}$, $x \neq -2$ and $x \neq 0$

Range: $y > 0$ and $y \leq -2$

c) Find he coordinates of all intersections of $f(x)$ with the line $y = 2x$.

[4 marks]

$$2x = \frac{2}{x^2+2x}$$

$$\Rightarrow 2x^3 + 4x^2 - 2 = 0$$

$$\Rightarrow 2(x+1)(x^2+x-1)$$

$$\Rightarrow x = -1, \quad x = \frac{-1}{2} - \frac{\sqrt{5}}{2}, \quad x = -\frac{1}{2} + \frac{\sqrt{5}}{2}$$

So coordinates of intersection points are

$$(-1, -2)$$

$$\left(\frac{-1}{2} - \frac{\sqrt{5}}{2}, -1 - \sqrt{5}\right)$$

$$\left(\frac{-1}{2} + \frac{\sqrt{5}}{2}, -1 + \sqrt{5}\right)$$

Section B

11 Let $X \sim N\left(63,10\right)$, then the standard deviation of X is

63 $\qquad$ $\sqrt{63}$ $\qquad$ 10 $\qquad$ $\sqrt{10}$

[1 mark]

12 Emily is wishing to investigate there political affiliation of the 3300 households in her local area.

She decides to obtain a sample of 150 households.

She enlists her friend, Lily, who suggests they select every 22nd household in a list of all the households until a sample of size 150 has been collected.

a) What is the population for this study.

The 150 households selected

All 3300 households

Lily and Emily

Houses that vote conservative.

[1 mark]

b) What is the sampling method suggested by Lily.

Systematic

Opportunity

Quota

Stratified

[1 mark]

13 A Head of Sixth Form is interested in the views of his students concerning Personal Development sessions in the Sixth Form.

A teacher suggests asking just Year 13 students as they have been in the Sixth Form for longer.

Why could this approach be problematic?

[2 marks]

This could lead to biased results as you aren't collecting the views of a sample of the whole population

14 John is playing a computer game where his character has to parachute into a particular landing zone.

From previous experience he knows that he has a probability of 0.4 of landing in the correct zone.

On a given day he plays the game 7 times.

a) Why is the binomial distribution a good model for this situation?

[2 marks]

Fixed number of times played.
Fixed probability of landing in the correct zone

b) Calculate the probability of landing on the zone in 4 or more of the seven games.

[2 marks]

Let $X \sim B(7, 0.4)$

$P(X \geq 4) = 1 - P(X \leq 3)$

$= 1 - 0.710208$

≈ 0.2898

c) If John plays the game 7 times on 3 consecutive days, what is the probability that he will land in the correct zone 4 or more times on each of the three days?

$0.2898^3 = 0.0243$

d) State one criticism of this approach used to model outcomes on consecutive days?

[1 mark]

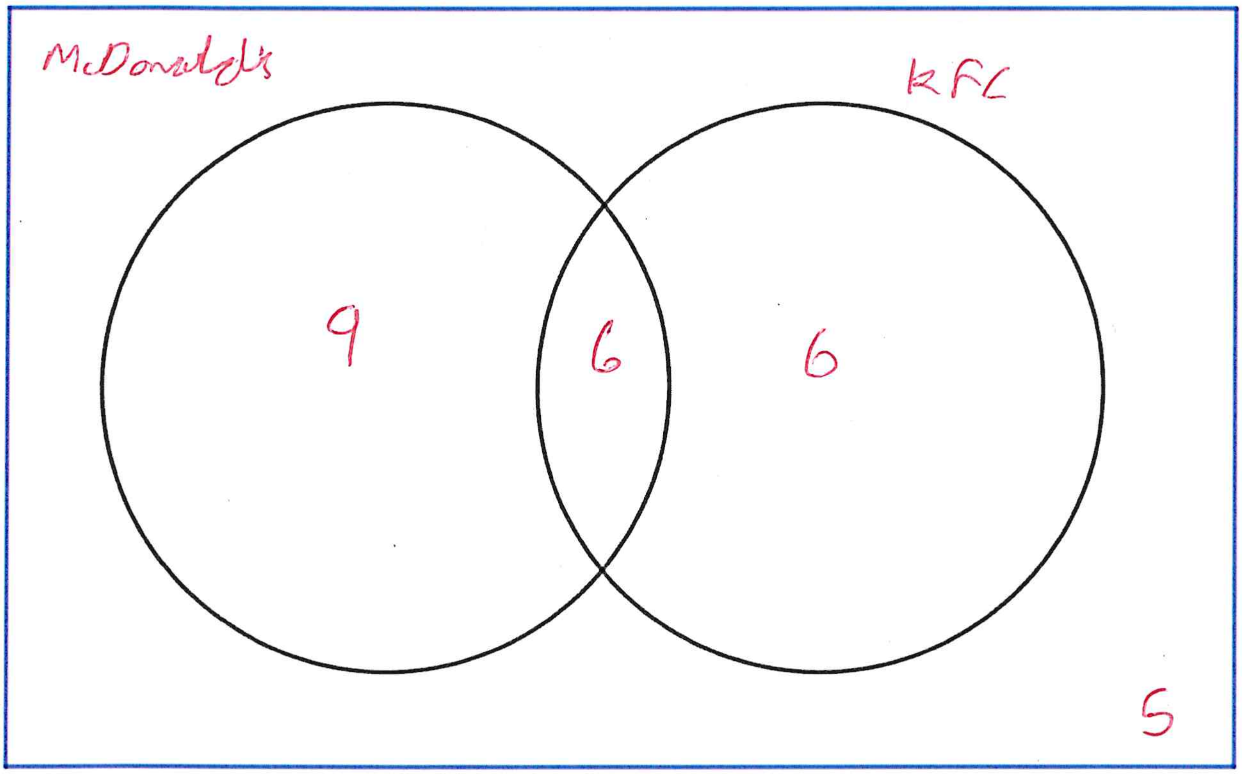
You would expect him to improve over time

15 A form tutor in Sixth Form asks her 26 students whether they like McDonalds and KFC. Of these 26,

- 15 like McDonalds
- 12 like KFC
- 6 like both McDonalds and KFC.

a) Complete the Venn diagram below.

[1 mark]



b) Determine whether liking KFC and liking McDonalds are independent events.

[3 marks]

$P(KFC \cap McD) = \frac{6}{26} = \frac{3}{13}$

$P(KFC) \times P(McD) = \frac{12}{26} \times \frac{15}{26}$

$= \frac{45}{169}$

Since $\frac{45}{169} \neq \frac{3}{13}$,

$P(KFC) \times P(McD) \neq P(KFC \cap McD)$, and so liking KFC and liking McDonalds are not independent events

c) In a larger survey it is established that

- The probability of a student liking fish and chips is $\frac{1}{4}$
- The probability of a student liking Thai food is $\frac{1}{3}$
- The probability of a student liking Thai food given that they like fish and chips is $\frac{2}{5}$.

Calculate the probability that a student likes fish and chips, or Thai food, or both.

[4 marks]

Let F = 'Likes Fish and Chips' and T = 'Like Thai'

$$P(F \cap T) = P(F) \times P(T|F)$$

$$= \frac{1}{4} \times \frac{2}{5}$$

$$= \frac{1}{16}$$

So,

$$P(F \cup T) = P(F) + P(T) - P(F \cap T)$$

$$= \frac{1}{4} + \frac{1}{3} - \frac{1}{10}$$

$$= \frac{29}{60}$$

16 Rupi is investigating car ownership in the UK and is using the AQA Large Data Set to do this.

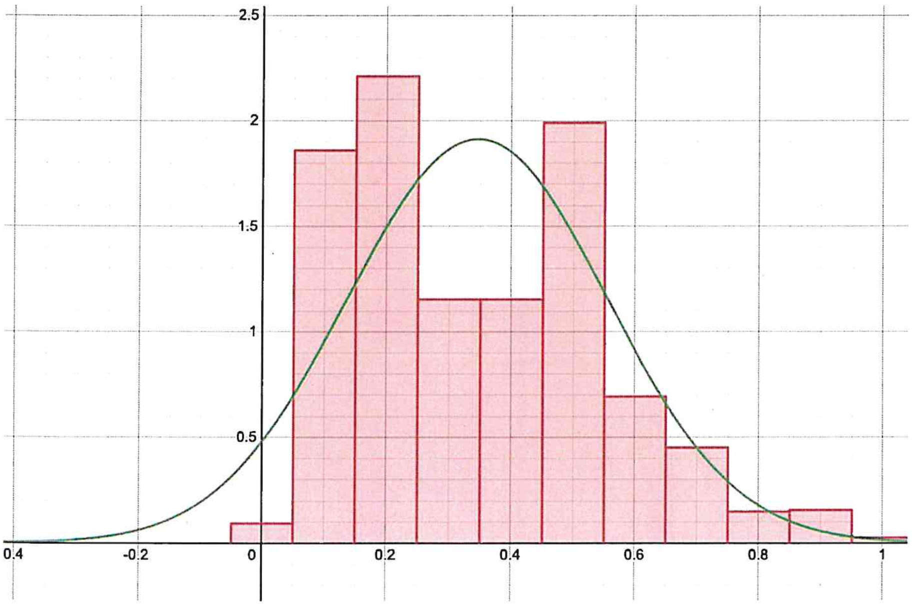
a) She concludes that Toyota is the 5th most popular car in the UK. Explain why she may be incorrect.

[2 marks]

The LDS doesn't include all makes of car and only includes the 5 most popular in two regions, so the national picture may be different

b) Rupi then looks at the CO emissions from the data. She fits a normal distribution to this data and then uses it to make inferences about the CO emissions across the whole UK.

With reference to the diagram below, which shows a histogram of CO emissions alongside the pdf of the fitted normal, evaluate her choice.



[3 marks]

The distribution of the sample data is not symmetrical, whereas a normal distribution is. Therefore the normal distribution is not a suitable example.
The level of CO emissions could also be different across the UK

17 Cholesterol levels for teenage boys are known to be approximately normally distributed with mean 170 and standard deviation 30.

A researcher is investigating the effects of a new diet regime on cholesterol levels.

a) Find the critical values, of the standard normal distribution at a $1\,\%$ level for the researcher who is interested in seeing if the diet has had an effect on cholesterol levels.

[2 marks]

This is a two tail test and so the 1% is split between the left and right tails.
So using the 0.005 percentage point of the standard normal distribution.

$$z_{crit} = \pm 2.58$$

b) A sample of 30 patients is tested and found to have a mean cholesterol level of 150.

Complete a hypothesis test to see if there is evidence of a change in cholesterol levels.

[5 marks]

Let

$$H_0 : \mu = 170$$

$$H_1 : \mu \neq 170$$

The appropriate test statistic is the sample mean $\bar{x} = 150$

Using the 1% significance level

$$\bar{X} \sim N\left(170, \frac{30^2}{30}\right)$$

Let
$$Z = \frac{\bar{X} - 170}{30/\sqrt{30}}$$

~~then~~

Critical regions are

$$\bar{x}_{crit} = -2.58\left(\frac{30}{\sqrt{30}}\right) + 170 = 155.868750$$

$$\bar{x}_{crit} = 2.58\left(\frac{30}{\sqrt{30}}\right) + 170 = 184.131247$$

Since $\bar{x} = 150 < 155.86$ there is sufficient evidence to reject H_0 in favour of H_1 and conclude that the diet has had an impact on cholesterol levels.

18 The monthly expenditure, $£P$, on petrol was recorded for 100 people and the following summary quantities computed.

$$\sum p = 8870, \qquad \sum p^2 = 790000$$

The maximum amount recorded was $£74.50$ and the minimum amount recorded was $£101.17$.

a) **i)** Find the mean, $\bar{p}$, of P

[1 mark]

$\bar{p} = \frac{8870}{100}$ A

$= £88.70$

ii) Find the standard deviation of the sample.

[2 marks]

$s.d = \sqrt{\frac{\sum x^2}{99} - \bar{x}^2} \approx 10.59$

or $s.d = \sqrt{\frac{\sum x^2}{100} - \bar{x}^2} = 5.68$

b) Using the results from **(a)** explain why a normal distribution could be a suitable model for the distribution of P.

[2 marks]

$\bar{p} + 3s \approx 120.5$

$\bar{p} - 3s \approx 56.4$

$\bar{p} + 3s \approx 105.74$

$\bar{p} - 3s \approx 71.66$

Since the maximal and minimal values recorded lie within $\bar{p} \pm 3s$ the Normal distribution is a suitable model

c) Assuming that P can be modelled by a normal distribution, using parameters calculated in **(a)** given to one decimal place, find

i) $\mathrm{P}(P = 70)$

[1 mark]

0

ii) $\mathrm{P}\left(82.50 \leq P \leq 96.20\right)$

[2 marks]

$= P(P \leq 96.20) - P(P \leq 82.50)$

$\approx 0.7605964261 - 0.2791199064$

≈ 0.4815

d The average monthly expenditure on diesel, £D, is known to be normally distributed with $D \sim \mathrm{N}\left(m, 8.5^2\right)$.

Given that $\mathrm{P}(D \leq 90) = 0.48$, find the value of m.

[4 marks]

$$P\left(Z = \frac{90-m}{8.5}\right) = 0.48$$

$$\Rightarrow \frac{90-m}{8.5} = 0.0501$$

$$\Rightarrow m = 90.4$$

19 A local company designs and organises the manufacture of lit up LED signs.

In the past $9\,\%$ of the signs received from the manufacturer are found to be faulty.

They change manufacturers and out of a sample of 40 signs, three are found to be faulty.

Determine whether, at the $5\,\%$ level of significance, there is sufficient evidence to conclude that the proportion of faulty signs has reduced.

[6 marks]

$H_0 : p = 0.09$

$H_1 : p < 0.09$

Under the null hypothesis

$X \sim B(40, 0.09)$

$P(X \leq 3) = 0.5092$

Since

$0.5092 > 0.05$ there is insufficient evidence to reject the null hypothesis, i.e. there is insufficient evidence to suggest the proportion of faulty signs has changed.