

AQA A-Level Maths 2022 Paper 3

Do not turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: **/ 100**



- 1 Given that $\frac{dy}{dx} = \frac{1}{8x^{\frac{3}{2}}}$ find an expression for y

$$y = -\frac{1}{4\sqrt{x}} + C$$

$$y = \frac{1}{4\sqrt{x}} + C$$

$$y = \frac{1}{12x^{\frac{5}{2}}} + C$$

$$y = -\frac{3}{16x^{\frac{5}{2}}} + C$$

[1 mark]

- 2 For what values of $|x|$ is the binomial expansion of $(2 + 5x)^{\frac{1}{3}}$ valid.

$$|x| < \frac{2}{5}$$

$$|x| < 1$$

$$|x| < 2$$

$$|x| < \frac{5}{2}$$

[1 mark]

- 3** The area between the curves $y = x^3$ and $y = x^{\frac{1}{3}}$ is M . What is the area between $y = x^3 + 2$ and $y = x^{\frac{1}{3}} + 2$?

$-M$

$M + 2$

M

$M - 2$

[1 mark]

- 4 a)** Let $f(x) = \frac{3x + 2}{x + 3}$. Find $f^{-1}(x)$.

[3 marks]

- b)** Why does the function $g(x) = x^2 + 4x + 8$, $-10 \leq x \leq 10$ not have an inverse.

[1 mark]

- 5** Prove, by contradiction, the arithmetic-geometric mean inequality,

$$\frac{1}{2}(a+b) \geq \sqrt{ab}, \quad a, b \geq 0, a \neq b$$

[5 marks]

- 6** Plastic balls are manufactured in such a way that the volume, V increases at a constant rate of $b \text{ cm}^3$ per second.
- a)** Find the rate of change of the surface area, A , of the ball in terms of r and b .

[4 marks]

- b)** If the rate of change of area is 2 cm^2 per second when the area is $4\pi \text{ cm}^2$, find the numerical value of the rate of change of the volume at this instant.

[2 marks]

- 7** The path of a rocket at time, t , seconds is modelled by the parametric equations

$$x = 10(2t)^{\frac{1}{2}}$$

$$y = 40t - 4t^2$$

- a)** What feature of these equations tells you that the rocket is not a projectile which is only acted upon by the force of gravity?

[1 mark]

- b)** Find the maximum height attained by the rocket.

[4 marks]

- c)** Explain, with justification why this model is not valid for $t \geq 10$.

[1 mark]

- 8 a)** A curve has equation $y = a \sin(x) + b \cos(x)$ where a and b are both positive constants.

The maximum value of y is 3 and the curve passes through the point $\left(\frac{\pi}{2}, \frac{3\sqrt{3}}{2}\right)$.

Find the exact values of a and b .

[4 marks]

- b)** Hence, find the solutions to $3\sqrt{3} \sin(x) + 3 \cos(x) = 0$ for $0 \leq x \leq 2\pi$.

[2 marks]

- 9** Geoff bought a classic car in 1996 for £9800.

A classic car specialist valued the car at 5 year intervals as shown in the table below.

Year	1996	2001	2006	2011
Value	9800	19500	39100	79000

The valuer suggests that the valuation price can be modelled by the equation $V = a \times b^t$, where t is the number of years after 1996.

- a)** Find the linearised form of the model given above.

[2 marks]

- b)** Complete the table below:

t	0	5	10	15
$\log_{10}(V)$				

c) By plotting a graph below, estimate the values of a and b .

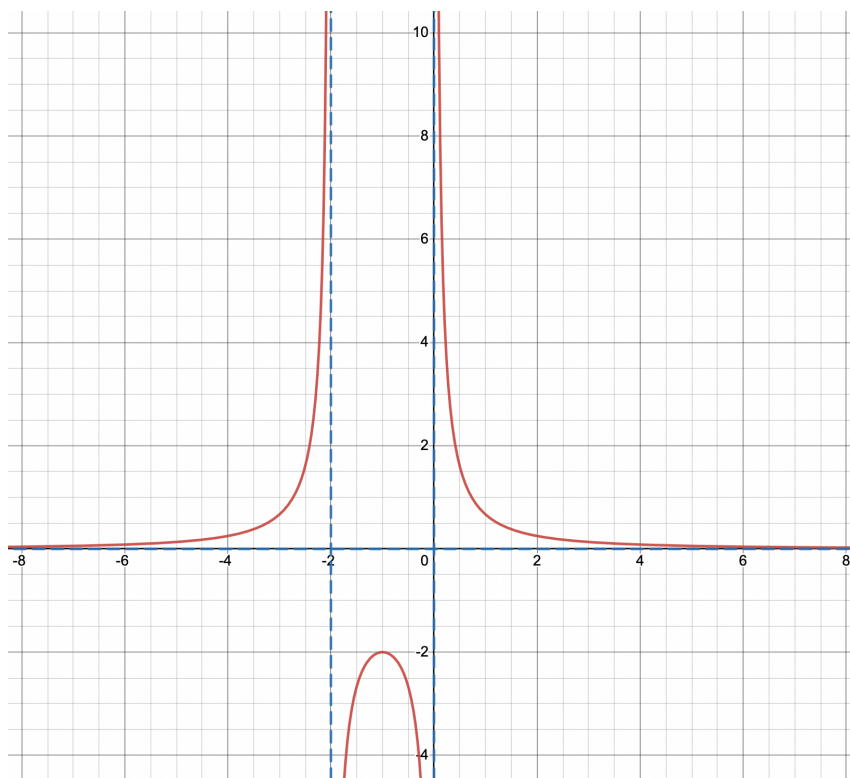
[4 marks]



- d)** Find the expected value of the car in 2032 and comment on the validity of this value.

[2 marks]

- 10 Consider $f(x) = \frac{2}{x^2 + 2x}$, the graph of which is shown below.



- a) Find the coordinates of the stationary point.

[4 marks]

- b)** Hence, state the domain and range of the function, $f(x)$.

[2 marks]

- c)** Find the coordinates of all intersections of $f(x)$ with the line $y = 2x$.

[4 marks]

Section B

- 11** Let $X \sim N(63, 10)$, then the standard deviation of X is

63 $\sqrt{63}$ 10 $\sqrt{10}$

[1 mark]

- 12** Emily is wishing to investigate there political affiliation of the 3300 households in her local area.

She decides to obtain a sample of 150 households.

She enlists her friend, Lily, who suggests they select every 22nd household in a list of all the households until a sample of size 150 has been collected.

- a)** What is the population for this study.

The 150 households selected

All 3300 households

Lily and Emily

Houses that vote conservative.

[1 mark]

- b)** What is the sampling method suggested by Lily.

Systematic

Opportunity

Quota

Stratified

[1 mark]

- 13** A Head of Sixth Form is interested in the views of his students concerning Personal Development sessions in the Sixth Form.

A teacher suggests asking just Year 13 students as they have been in the Sixth Form for longer.

Why could this approach be problematic?

[2 marks]

- 14** John is playing a computer game where his character has to parachute into a particular landing zone.

From previous experience he knows that he has a probability of 0.4 of landing in the correct zone.

On a given day he plays the game 7 times.

- a)** Why is the binomial distribution a good model for this situation?

[2 marks]

- b)** Calculate the probability of landing on the zone in 4 or more of the seven games.

[2 marks]

- c)** If John plays the game 7 times on 3 consecutive days, what is the probability that he will land in the correct zone 4 or more times on each of the three days?

- d)** State one criticism of this approach used to model outcomes on consecutive days?

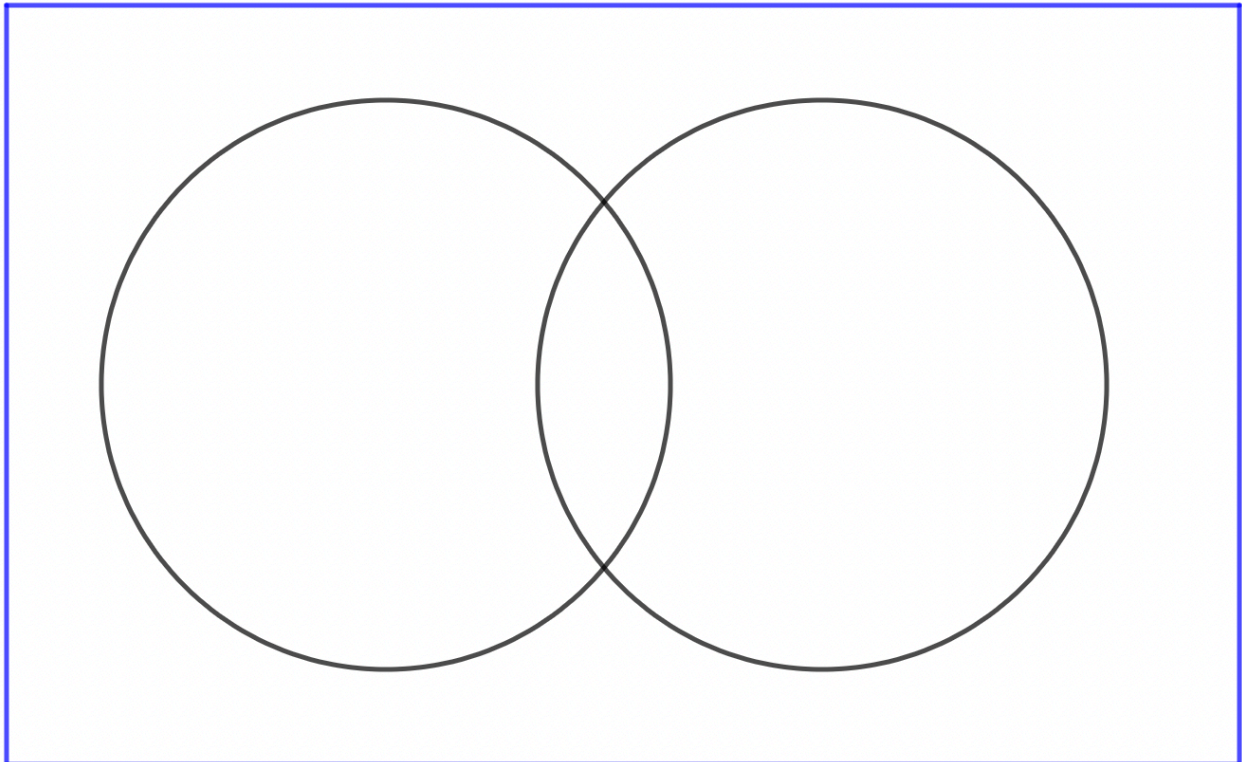
[1 mark]

15 A form tutor in Sixth Form asks her 26 students whether they like McDonalds and KFC. Of these 26,

- 15 like McDonalds
- 12 like KFC
- 6 like both McDonalds and KFC.

a) Complete the Venn diagram below.

[1 mark]



b) Determine whether liking KFC and liking McDonalds are independent events.

[3 marks]

c) In a larger survey it is established that

- The probability of a student liking fish and chips is $\frac{1}{4}$
- The probability of a student liking Thai food is $\frac{1}{3}$
- The probability of a student liking Thai food given that they like fish and chips is $\frac{2}{5}$.

Calculate the probability that a student likes fish and chips, or Thai food, or both.

[4 marks]

16 Rupi is investigating car ownership in the UK and is using the AQA Large Data Set to do this.

- a)** She concludes that Toyota is the 5th most popular car in the UK. Explain why she may be incorrect.

[2 marks]

- b)** Rupi then looks at the *CO* emissions from the data. She fits a normal distribution to this data and then uses it to make inferences about the *CO* emissions across the whole UK.

With reference to the diagram below, which shows a histogram of *CO* emissions alongside the pdf of the fitted normal, evaluate her choice.



[3 marks]

- 17** Cholesterol levels for teenage boys are known to be approximately normally distributed with mean 170 and standard deviation 30.

A researcher is investigating the effects of a new diet regime on cholesterol levels.

- a)** Find the critical values, of the standard normal distribution at a 1 % level for the researcher who is interested in seeing if the diet has had an effect on cholesterol levels.

[2 marks]

- b)** A sample of 30 patients is tested and found to have a mean cholesterol level of 150.

Complete a hypothesis test to see if there is evidence of a change in cholesterol levels.

[5 marks]

- 18** The monthly expenditure, £ P , on petrol was recorded for 100 people and the following summary quantities computed.

$$\sum p = 8870, \quad \sum p^2 = 790000$$

The maximum amount recorded was £74.50 and the minimum amount recorded was £101.17.

- a) i)** Find the mean, \bar{p} , of P

[1 mark]

- ii)** Find the standard deviation of the sample.

[2 marks]

- b)** Using the results from **(a)** explain why a normal distribution could be a suitable model for the distribution of P .

[2 marks]

- c)** Assuming that P can be modelled by a normal distribution, using parameters calculated in **(a)** given to one decimal place, find

i) $P(P = 70)$

[1 mark]

ii) $P(82.50 \leq P \leq 96.20)$

[2 marks]

- d** The average monthly expenditure on diesel, $\pounds D$, is known to be normally distributed with $D \sim N(m, 8.5^2)$.

Given that $P(D \leq 90) = 0.48$, find the value of m .

[4 marks]

- 19** A local company designs and organises the manufacture of lit up LED signs.

In the past 9 % of the signs received from the manufacturer are found to be faulty.

They change manufacturers and out of a sample of 40 signs, three are found to be faulty.

Determine whether, at the 5 % level of significance, there is sufficient evidence to conclude that the proportion of faulty signs has reduced.

[6 marks]