

# **AQA A-Level Maths 2022 Paper 2**

Do not turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

**Name:**

**Total Marks:** / 100



- 1 The centre and radius of the circle  $x^2 - 6x + y^2 + 4y - 1 = 0$  are:

Centre :  $(3, - 2)$

Radius :  $\sqrt{14}$

Centre :  $(3, - 2)$

Radius :  $\sqrt{13}$

Centre :  $(-3, 2)$

Radius :  $\sqrt{14}$

Centre :  $(-3, 2)$

Radius :  $\sqrt{13}$

**[1 mark]**

- 2 Write as a single logarithm  $5 \log(x) + \log(y^3z^6) - \log(xz)$

$\log(yz)$

$\log(x^4y^3z^5)$

$\log(x^3y^3z^4)$

$\log(x^5 + y^3z^6 - xz)$

**[1 mark]**

- 3 a) Prove that 53 is a prime number.

[2 marks]

$$\sqrt{53} \approx 7.28$$

$$53 \div 2 \not\in \mathbb{Z}$$

$$53 \div 3 \not\in \mathbb{Z}$$

$$53 \div 5 \not\in \mathbb{Z}$$

$$53 \div 7 \not\in \mathbb{Z}$$

Hence  $53 \in \text{prime}$

- b) Disprove the statement "For  $n \in \mathbb{N}$ ,  $n^3 + 3n - 5$  always prime".

[2 marks]

$$\begin{aligned} \text{When } n = 5, n^3 + 3n - 5 &= 5^3 + 3 \times 5 - 5 \\ &= 125 \\ &= 5 \times 27 \end{aligned}$$

- 4 a) Find and classify the stationary points of the polynomial

$$p(x) = 2x^3 + 3x^2 - 72x + 18$$

[5 marks]

$$p'(x) = 6x^2 + 6x - 72$$

$$= 6(x^2 + x - 12)$$

at a stationary point  $p'(x) = 0$

$$\Rightarrow 0 = 6(x^2 + x - 12)$$

$$= 6(x-3)(x+4)$$

$$\Rightarrow x = 3 \text{ or } x = -4$$

$$\text{when } x = 3, y = -117$$

$$\text{when } x = -4, y = 226$$

$$\text{d}^2 p(x) = 12x + 6$$

$$\text{when } x = 3, p''(3) > 0$$

$$\text{when } x = -4, p''(-4) < 0$$

Hence,  $(3, -117)$  is a local minimum and  $(-4, 226)$  is a local maximum.

- b) What would the stationary points of  $f(x + 2) + 3$  be?

[2 marks]

$$(1, -114) \text{ and } (-6, 229)$$

- 5 Joshua is trying to find the derivative of  $y = \frac{1}{x}$  by first principles.

He begins by writing:

$$\begin{aligned}\frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{-h}{hx + h^2}\end{aligned}$$

- a) Identify the mathematical errors Joshua has made.

[2 marks]

He hasn't performed the fraction subtraction correctly as he hasn't put them over a common denominator

- b) Write a complete, rigorous proof for the derivation of the derivative of  $y = \frac{1}{x}$  from first principles.

$$\text{Let } f(x) = \frac{1}{x},$$

Then, applying the definition

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-h}{x(x+h)h}$$

$$= \lim_{h \rightarrow 0} -\frac{1}{x(x+h)}$$

$$= \lim_{h \rightarrow 0} -\frac{1}{x^2 + xh}$$

$$= -\frac{1}{x^2}$$

- 6 a) Find the first three terms, in ascending powers of  $x$ , of the binomial expansion of  $\frac{1}{\sqrt[3]{8+2x}}$

$$\frac{1}{\sqrt[3]{8+2x}} = (8+2x)^{-\frac{1}{3}} = \frac{1}{2} \left(1 + \frac{1}{4}x\right)^{-\frac{1}{3}} \quad [3 \text{ marks}]$$

$$= \frac{1}{2} \left[ 1 + \left(-\frac{1}{3}\right)\left(\frac{x}{4}\right) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2} \left(\frac{x}{4}\right)^2 + \dots \right]$$

$$= \frac{1}{2} - \frac{x}{24} + \frac{x^2}{144}$$

- b) Hence, find the expansion of  $\frac{1}{\sqrt[3]{8-2x^2}}$

[2 marks]

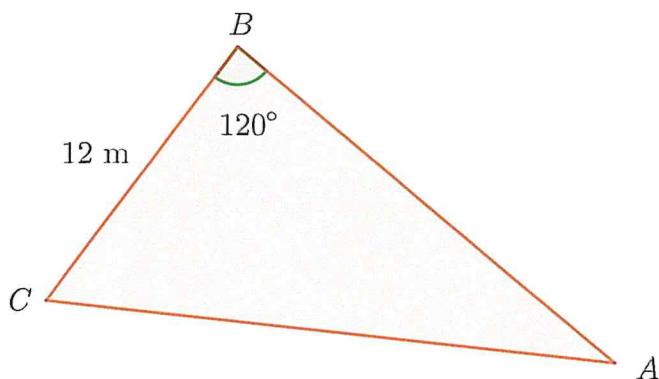
$$\frac{1}{2} - \frac{x^2}{24} + \frac{x^4}{144}$$

- c) Millie uses the first three terms of the expansion found in (b) to find an approximation to the integral  $\int_0^{\frac{1}{2}} \frac{2}{\sqrt[3]{8 - 2x^2}} dx$ . Evaluate this approximation.

[3 marks]

$$\begin{aligned}
 \int_0^{\frac{1}{2}} \frac{2}{\sqrt[3]{8 - 2x^2}} dx &= 2 \int_0^{\frac{1}{2}} \frac{1}{\sqrt[3]{8 - 2x^2}} dx \\
 &= 2 \int_0^{\frac{1}{2}} \left( \frac{1}{2} - \frac{x^2}{24} + \frac{x^4}{144} \right)^{-\frac{1}{3}} dx \\
 &= 2 \left[ \frac{1}{2}x - \frac{x^3}{72} + \frac{x^5}{720} \right]_0^{\frac{1}{2}} \\
 &= 2 \left[ \left( \frac{1}{2} \times \frac{1}{2} - \frac{(\frac{1}{2})^3}{72} + \frac{(\frac{1}{2})^5}{720} \right) - (0) \right] \\
 &= 2 \times \frac{1907}{7680} \\
 &= \frac{1907}{3840} \\
 &\approx 0.4966 (4 s.f.) \\
 &= 0.497 \text{ to } 3 \text{ dp}
 \end{aligned}$$

- 7 Consider the triangle  $ABC$  of area  $72 \text{ m}^2$ .



- a) Find the length of  $AB$ .

[2 marks]

Since  $\text{Area} = \frac{1}{2}ab\sin(C)$  for the triangle,

$$72 = \frac{1}{2} \times 12 \times |BA| \times \sin(120^\circ)$$

$$\Rightarrow |AB| = \frac{72 \times 2}{12 \times \frac{\sqrt{3}}{2}}$$

$$= 8\sqrt{3}$$

- b) Find the length  $AC$ .

Using the cosine rule

$$|AC|^2 = |BC|^2 + |AB|^2 - 2|BC||AB|\cos(120^\circ)$$

$$= 12^2 + (8\sqrt{3})^2 - 2 \times 12 \times 8\sqrt{3} \times \cos(120^\circ)$$

$$= 502.2768775$$

$$\Rightarrow |AC| = 22.41133 \text{ m}$$

$$= 22.4 \text{ m}$$

c) Hence, find the smallest angle in the triangle.

[2 marks]

$$\frac{\sin(C)}{8\sqrt{3}} = \frac{\sin(120)}{|AC|}$$

$$\Rightarrow \sin(C) = \frac{\sin(120)}{|AC|} \times \frac{8\sqrt{3}}{1}$$

$$\Rightarrow C = \arcsin\left(\frac{8\sqrt{3}\frac{\sqrt{3}}{2}}{|AC|}\right)$$

$$= \arcsin\left(\frac{12}{|AC|}\right)$$

$$= 32.37365913$$

$$= 32.4^\circ$$

But this isn't the smallest angle in the triangle,

$$180 - 120 - 32.373... = 27.6264087$$

so the smallest angle is approximately  $27.6^\circ$

- 8 The differential equation below models the concentration of a reagent in a chemical reaction. When  $t = 0$  seconds the scaled concentration takes a value of 2. Find the value of the scaled concentration when  $t = 2$  seconds.

$$\frac{dy}{dt} = \frac{e^{-y}(5t+8)}{t^2+3t+2}$$

[8 marks]

Separating the variables,

$$\int \frac{1}{e^{-y}} \frac{dy}{dt} dt = \int \frac{5t+8}{t^2+3t+2} dt$$

$$\Rightarrow \int e^y dy = \int \frac{3}{t+1} + \frac{2}{t+2} dt$$

$$\Rightarrow e^y = 3 \ln(t+1) + 2 \ln(t+2) + C$$

When  $t=0$ ,  $y=2$ , hence

$$e^2 = 0 + 2 \ln(2) + C \Rightarrow C = e^2 - 2 \ln(2)$$

So

$$e^y = 3 \ln(t+1) + 2 \ln(t+2) + e^2 - 2 \ln(2)$$

$$\Rightarrow y = \ln(3 \ln(t+1) + 2 \ln(t+2) + e^2 - 2 \ln(2))$$

When

$$t=2, \quad y \approx 2.41$$

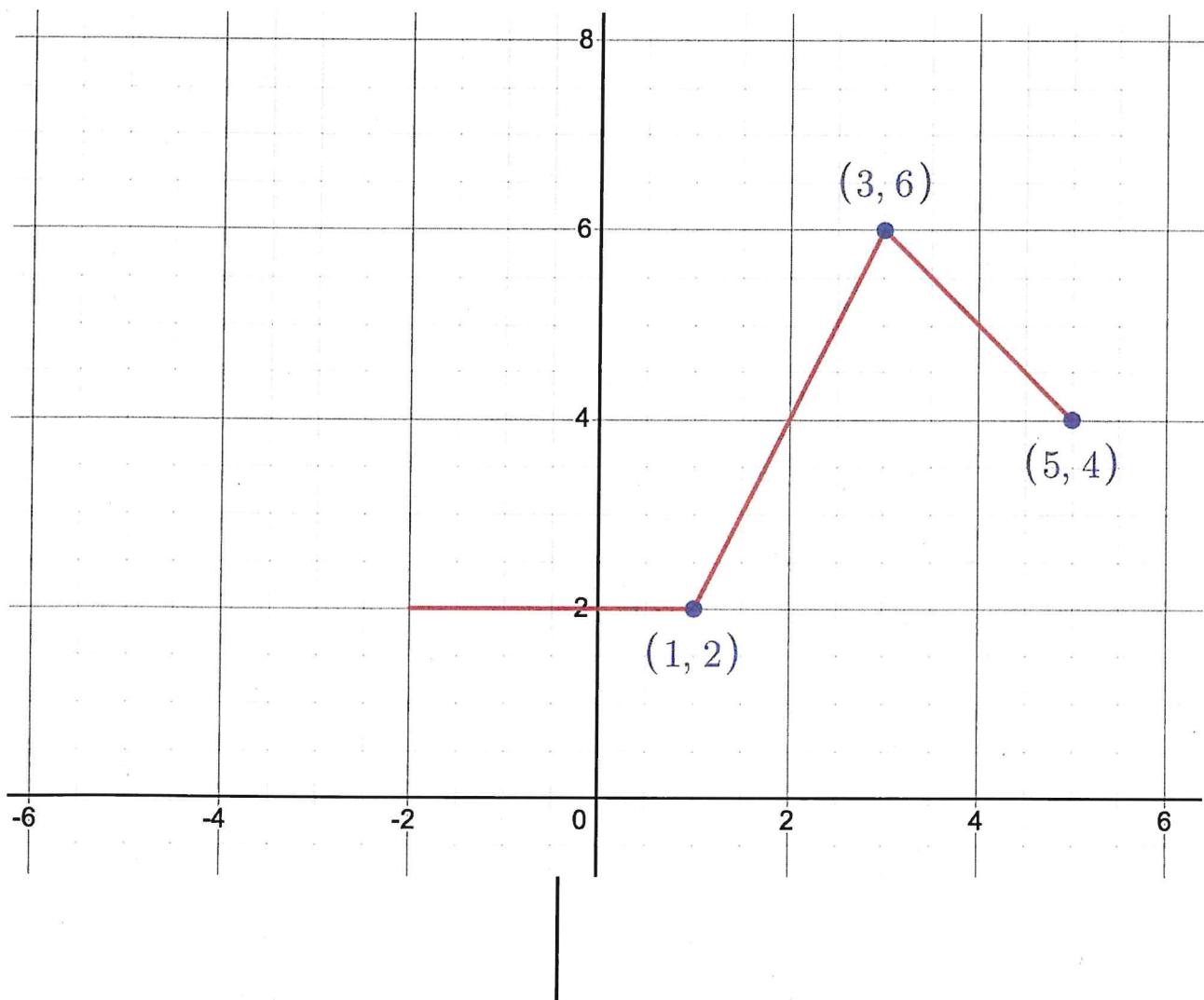
$$\begin{aligned} \text{or exactly } y &= \ln(3\ln(3) + 2\ln(4) + e^2 - 2(\ln(2))) \\ &= \ln(\ln(27) + 2\ln(4) + e^2 - \ln(4)) \\ &= \ln(\ln(27) + \ln(16) + e^2) \end{aligned}$$

- 9 a) Sketch the curve defined by the following

$$f(x) = \begin{cases} 2, & -2 \leq x \leq 1 \\ 2x, & 1 \leq x \leq 3 \\ -x + 9 & 3 \leq x \leq 5 \end{cases}$$

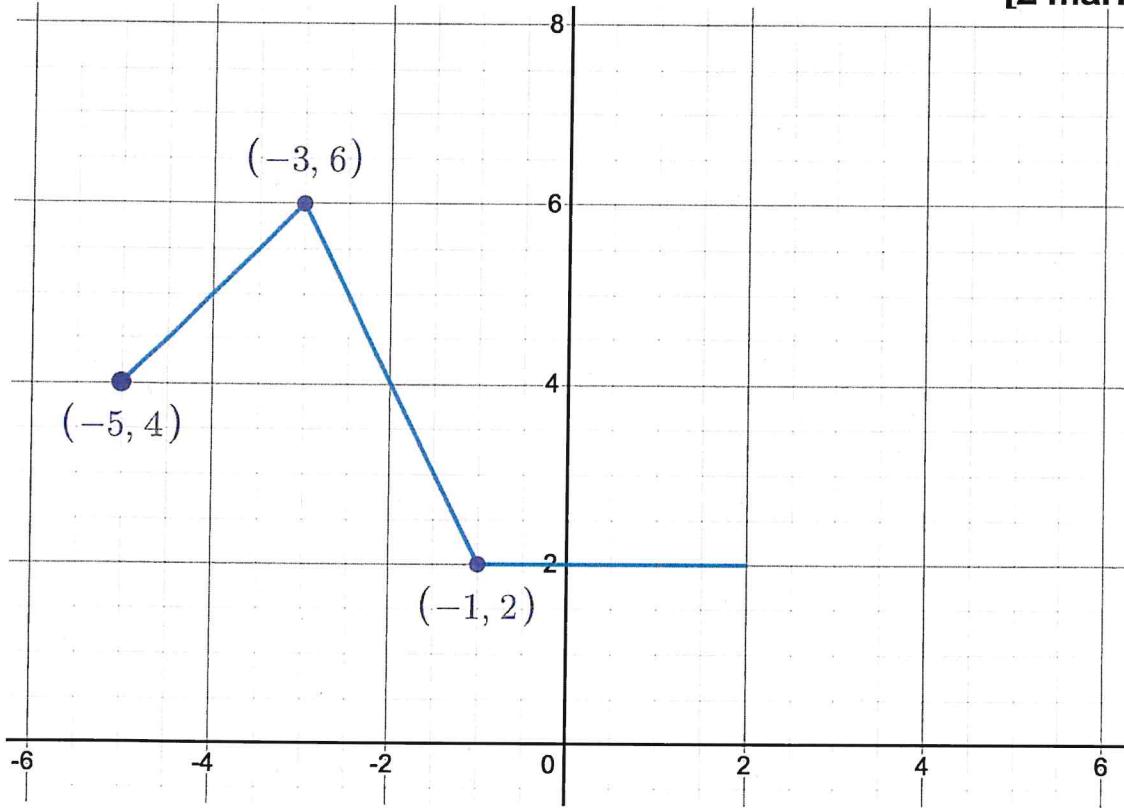
Label the points where  $x = 1$ ,  $x = 3$  and  $x = 5$ .

[2 marks]



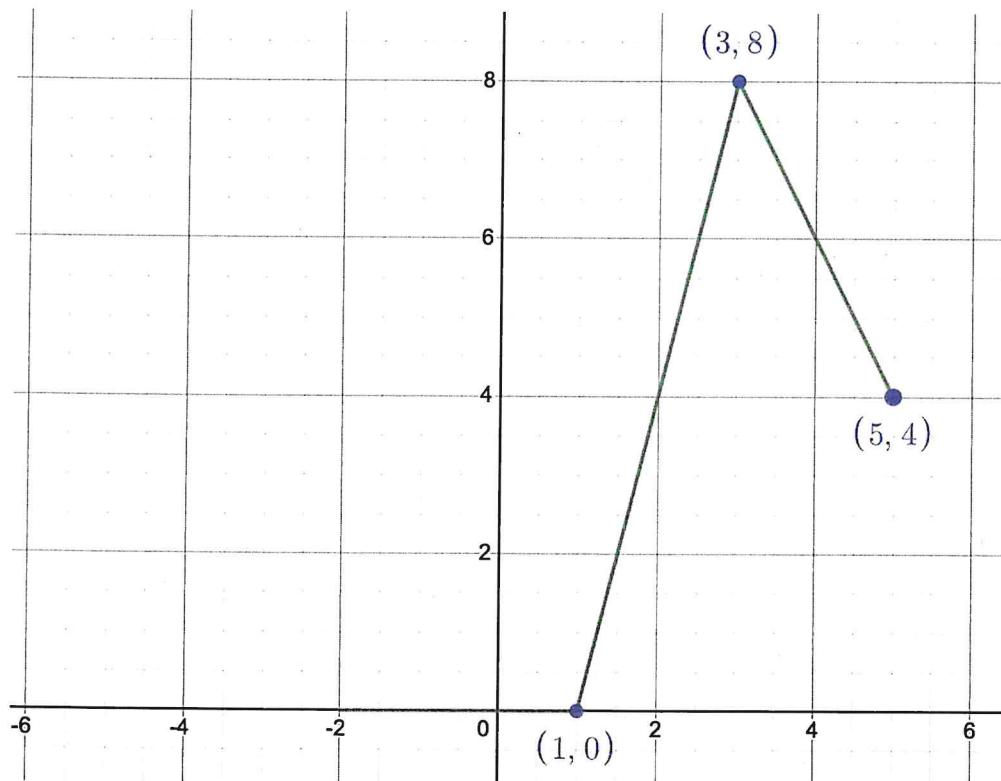
- b) Hence, sketch  $f(-x)$ , showing the coordinates of the transformations of the points labelled in (a).

[2 marks]



- c) Sketch  $2f(x) - 4$ , showing the coordinates of the transformations of the points labelled in (a).

[2 marks]



- 10 a) Find the points of inflection for the curve  $y = e^{-2x^2+x}$ .

[5 marks]

$$y = e^{-2x^2+x}$$

$$\frac{dy}{dx} = (1 - 4x) e^{-2x^2}$$

$$\frac{d^2y}{dx^2} = (16x^2 - 8x - 3) e^{-2x^2}$$

At inflection point  $\frac{d^2y}{dx^2} = 0$ , so

$$16x^2 - 8x - 3 = 0$$

$$\Rightarrow (4x+1)(4x-3) = 0$$

$$\Rightarrow x = -\frac{1}{4} \text{ and } x = \frac{3}{4} \text{ at inflection}$$

points.

- b) Hence, find the regions where  $f(x)$  is concave and convex.

[2 marks]

Concave if  $\frac{d^2y}{dx^2} < 0$

$$\Rightarrow -\frac{1}{4} \leq x \leq \frac{3}{4}$$

Convex if  $\frac{d^2y}{dx^2} > 0$

$$x < -\frac{1}{4} \text{ and } x > \frac{3}{4}$$

## Section B

- 11** A number of forces act on a particle such that there resultant force is  
 $\begin{pmatrix} -4 \\ 10 \end{pmatrix}$  N.

One of the forces is  $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$  N. Calculate the total of the other forces acting on the particle.

$$\begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

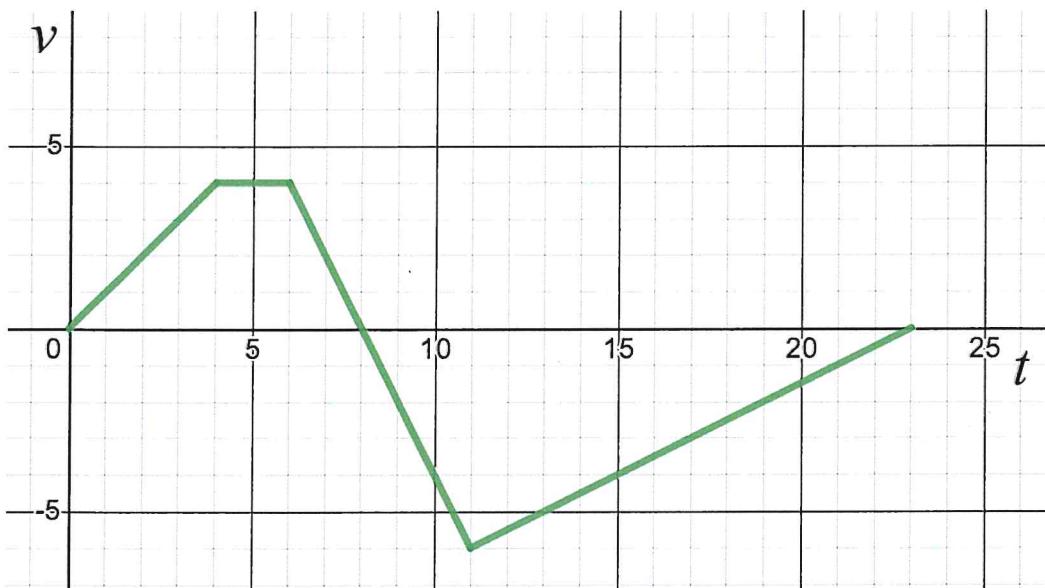
$$\begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} -6 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} -2 \\ 12 \end{pmatrix}$$

[1 mark]

- 12** The velocity time graph below shows the velocity of a particle against time.



The total distance travelled is

0 m

-25 m

65 m

25 m

[1 mark]

- 13 A particle of mass 3 kg moves in a horizontal plane under the action of a resultant force  $\mathbf{F}$  newtons. The velocity of the particle is

$$\mathbf{v} = \begin{pmatrix} 18t + \cos(t) \\ e^{-t} + \sin(t) \end{pmatrix}$$

- a) Find an expression for the acceleration  $\mathbf{a}$  as a function of  $t$ .

[2 marks]

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \begin{pmatrix} 18 - \sin(t) \\ -e^{-t} + \cos(t) \end{pmatrix}$$

- b) (i) Find the magnitude of  $\mathbf{F}$  at  $t = 0$ .

[2 marks]

$$\text{Apply } \mathbf{F} = m\mathbf{a}, \text{ so}$$

$$\mathbf{F} = 3 \begin{pmatrix} 18 - \sin(0) \\ -e^0 + \cos(0) \end{pmatrix}$$

At  $t = 0$

$$\mathbf{F} = \begin{pmatrix} 3 \times 18 \\ 3 \times (-1+1) \end{pmatrix} = \begin{pmatrix} 54 \\ 0 \end{pmatrix} \text{ N}$$

$\therefore$  the magnitude of  $\mathbf{F}$  is 54 N

- (ii) In what direction is the force acting at this time?

[1 mark]

Due east.

- c) When  $t = 0$  the particle is at the point with position vector  $5\mathbf{i} + 2\mathbf{j}$ .

Find the position vector  $\mathbf{r}$  of the particle at time  $t$ .

[4 marks]

$$\begin{aligned}\mathbf{r} &= \left( \begin{array}{l} \int_0^t (8t - \cos(t)) dt \\ \int_0^t e^{-t} + \sin(t) dt \end{array} \right) \\ &= \left( \begin{array}{l} 9t^2 + \sin(t) + C_1 \\ -e^{-t} - \cos(t) + C_2 \end{array} \right)\end{aligned}$$

When  $t = 0$ , the particle is at the point with position vector  $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ ,

hence  $\begin{pmatrix} 5 \\ 2 \end{pmatrix} = \left( \begin{array}{l} 9 \times 0^2 + \sin(0) + C_1 \\ -e^0 - \cos(0) + C_2 \end{array} \right)$

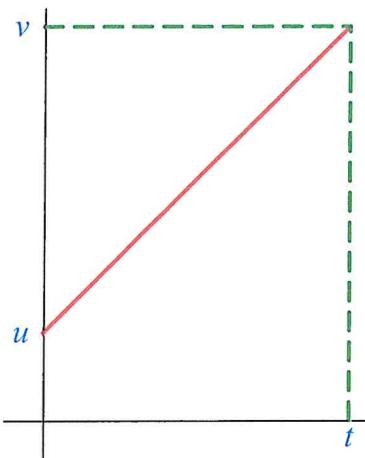
$$\Rightarrow \begin{aligned}C_1 &= 5 \\ C_2 &= 2\end{aligned}$$

and so,

$$\mathbf{r} = \left( \begin{array}{l} 9t^2 + \sin(t) + 5 \\ -e^{-t} - \cos(t) + 2 \end{array} \right)$$

14

Using the graph shown below, or otherwise, show that  $v^2 = u^2 + 2as$ .



Standard bookwork.

[4 marks]

Displacement is the area under the graph. Considering the area of a trapezium

$$s = \frac{1}{2}(u+v)t \quad (1)$$

Acceleration is the gradient,

$$\frac{v-u}{t} = a \Rightarrow t = \frac{v-u}{a} \quad (2)$$

Substitute (2) into (1)

$$s = \frac{1}{2}(u+v)\left(\frac{v-u}{a}\right)$$

$$\Rightarrow 2as = (u+v)(v-u)$$

$$2as = v^2 - u^2$$

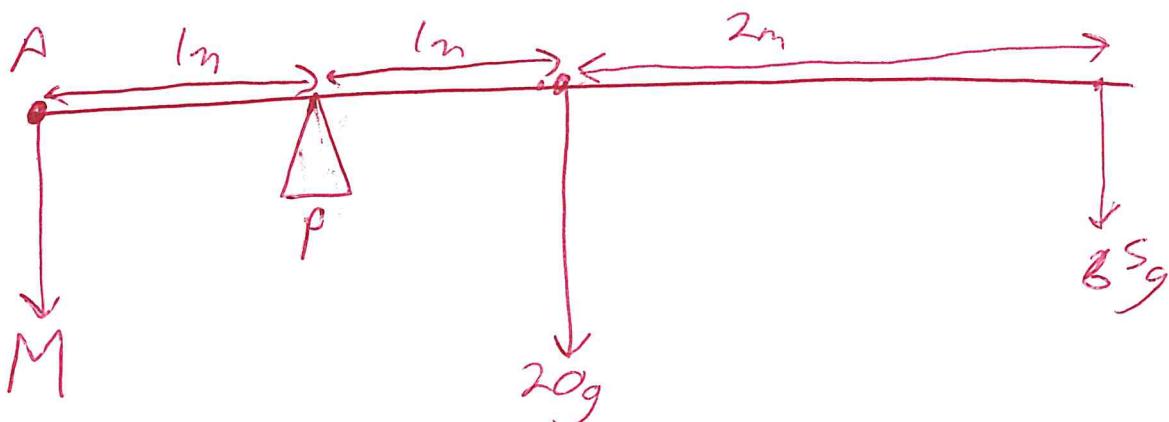
$$\Rightarrow v^2 = u^2 + 2as$$

- 16 A uniform plank,  $AB$ , of length 4 m and mass 20 kg is balanced on a pivot,  $P$ , which is 1 m away from  $A$ .

A mass of  $M$  kg is placed at  $A$  and a lady of mass 65 kg stands on the plank at  $B$ .

Given that the plank is in equilibrium, with neither  $A$  nor  $B$  touching the ground, find the value of  $M$ .

[3 marks]



$$\text{At } P: M \times 1 = 1 \times 20 + 3 \times 65$$

$$\begin{aligned} \Rightarrow M &= 20 + 3 \times 65 \\ &= 215 \text{ kg} \end{aligned}$$

- 17 A particle  $A$  moves on a horizontal surface with constant acceleration  $-0.2\mathbf{i} + 0.1\mathbf{j}$  ms $^{-2}$  and starts at the origin with velocity  $3\mathbf{i} + 4\mathbf{j}$  ms $^{-1}$ .

- a) Find the position vector,  $\mathbf{r}$ , of  $A$   $t$  seconds after leaving the origin.

$$\begin{aligned}\mathbf{r}_A &= \begin{pmatrix} 3 \\ 4 \end{pmatrix}t + \frac{1}{2} \begin{pmatrix} -0.2 \\ 0.1 \end{pmatrix} t^2 && \text{using } s = ut + \frac{1}{2}at^2 \quad [2 \text{ marks}] \\ &= \begin{pmatrix} 3t - 0.1t^2 \\ 4t + 0.05t^2 \end{pmatrix}\end{aligned}$$

- b) Find the time it takes for the particle to reach a point due north of the origin.

[3 marks]

Due north when the  $\mathbf{i}$  component is zero

$$3t - 0.1t^2 = 0$$

$$\Rightarrow t(3 - 0.1t) = 0$$

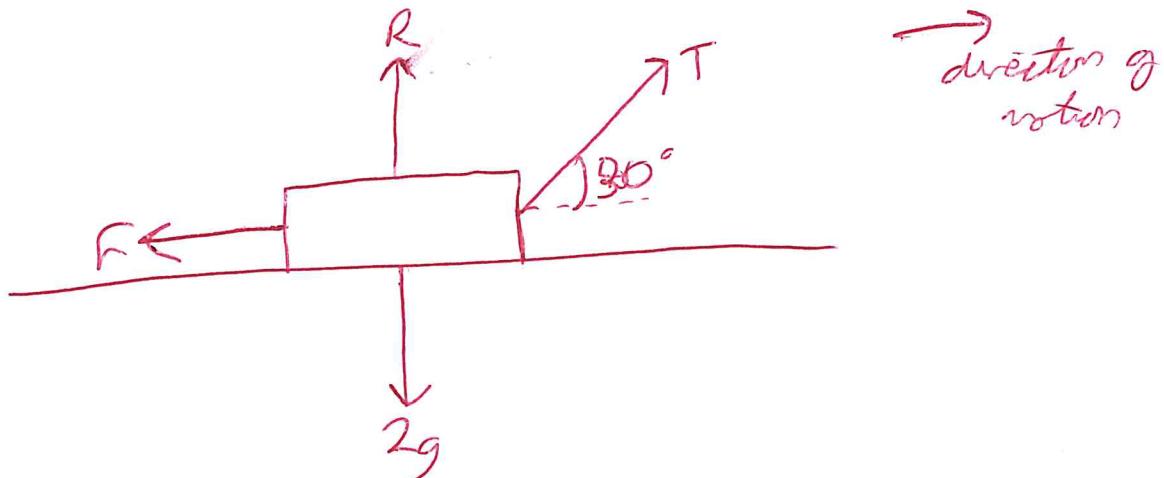
$$3 = 0.1t$$

$$\Rightarrow t = 30 \text{ seconds}$$

- 18 A block, of mass 2 kg is pulled across a rough horizontal surface by a force of 20 N inclined at an angle of  $30^\circ$  to the horizontal. The coefficient of friction is 0.6.

a) Draw a force diagram of this situation.

[2 marks]



- b) Find the acceleration of the block when it is under the action of this force.

[4 marks]

$$\begin{aligned} R \uparrow \quad R + 20 \sin(30) &= 2g \\ R &= 2g - 20 \sin(30) \\ &= 11.4 \text{ N} \end{aligned}$$

$$\text{Then } F = \mu R = \frac{11.4}{2} \times 0.6 = 3.42 \text{ N}$$

$$\text{Apply } F = ma \rightarrow$$

$$20 \cos(30) - F = ma$$

$$\Rightarrow a = \frac{20 \cos(30) - F}{2}$$

$$-\frac{114}{2s} = 2a$$

$$\Rightarrow a = -\frac{114}{50}$$

Then

$$S = -$$

$$U = 17.34$$

$$V = 0$$

$$a = -114/50$$

$$t =$$

So using  $V = u + at$

$$0 = 17.34 - \frac{114}{50} t$$

$$\Rightarrow t = \frac{17.34 \times 50}{114}$$

$\approx 7.6$  seconds.

- 19 A ball is projected from the ground with speed  $u$  at an angle  $\alpha$  to the horizontal where the only force acting on the ball is gravity.

The  $x$ - and  $y$ -axes are horizontal and vertical, passing through the origin  $O$  in the plane of motion of the ball.

- a) Find the time of flight of the particle.

	Horizontal motion	Vertical motion	[4 marks]
$s$	$s_H$	$0$	
$u$	$u \cos \alpha$	$u \sin \alpha$	
$v$	$u \cos \alpha$		
$a$	$0$	$-g$	
$t$	$T$	$T$	

Considering horizontal motion,

$$s_H = u T \cos(\alpha)$$

Considering vertical motion,

$$0 = u T \sin(\alpha) - \frac{1}{2} g T^2$$

$$= T(u \sin(\alpha) - \frac{1}{2} g T)$$

$s_0$

$$T = 0 \text{ or } T = \frac{2 u \sin(\alpha)}{g}$$

b) Show that the equation of the path can be given by

$$y = x \tan(\alpha) - \frac{gx^2}{2u^2} (1 + \tan^2(\alpha))$$

[6 marks]

	Horizontal	Vertical
s	x	y
v	$u \cos(\alpha)$	$u \sin(\alpha)$
v	$u \cos(\alpha)$	
a	0	-g
t	T	T

Considering horizontal motion

$$x = ut \cos(\alpha) \quad ①$$

Considering vertical motion

$$y = ut \sin(\alpha) - \frac{1}{2}gt^2 \quad ②$$

From ①  $t = \frac{x}{u \cos(\alpha)} \quad ③$

Substituting ③ into ②

$$y = u \frac{x}{u \cos(\alpha)} \sin(\alpha) - \frac{1}{2} g \left( \frac{x}{u \cos(\alpha)} \right)^2$$

$$= x \tan(\alpha) - \frac{gx^2}{2u^2 \cos^2(\alpha)}$$

$$= x \tan(\alpha) - \frac{gx^2}{2u^2} \sec^2(\alpha)$$

$$= x \tan(\alpha) - \frac{gx^2}{2u^2} (1 + \tan^2(\alpha))$$

c) Find the range of the particle.

[3 marks]

The range is the value of  $x$  when

$$t = \frac{2u \sin(\alpha)}{g}$$
 from (a)

$$\Rightarrow \text{using } s = ut \cos(\alpha)$$

$$x = u \frac{2u \sin(\alpha) \cos(\alpha)}{g}$$

$$= \frac{2u^2 \sin(\alpha) \cos(\alpha)}{g}$$

$$= \frac{u^2 \sin(2\alpha)}{g}$$

Not asked for but the range is maximised when

$$\sin(2\alpha) = 1 \Rightarrow \alpha = 45^\circ \text{ when the range is } \frac{u^2}{g}$$