AQA A-Level Maths 2022 Paper 2

Do not turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: / 100

1 The centre and radius of the circle $x^2 - 6x + y^2 + 4y - 1 = 0$ are:

Centre : $(3, -2)$ Radius : $\sqrt{14}$

Centre : $(3, -2)$ Radius : $\sqrt{13}$

Centre : $(-3,2)$ Radius : $\sqrt{14}$

Centre : $(-3,2)$ Radius : $\sqrt{13}$

[1 mark]

2 Write as a single logarithm $5\log(x) + \log\left(y^3z^6\right) - \log(xz)$

$\log(yz)$

$\log\left(x^4y^3z^5\right)$

$\log\left(x^3y^3z^4\right)$

$\log\left(x^5 + y^3z^6 - xz\right)$

[1 mark]

3 **a)** Prove that 53 is a prime number.

[2 marks]

b) Disprove the statement "For $n \in \mathbb{N}$, $n^3 + 3n - 5$ always prime".

[2 marks]

4 **a)** Find and classify the stationary points of the polynomial

$$p(x) = 2x^3 + 3x^2 - 72x + 18$$

[5 marks]

b) What would the stationary points of $f(x + 2) + 3$ be?

[2 marks]

5 Joshua is trying to find the derivative of $y = \frac{1}{x}$ by first principles.

He begins by writing:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$
$$= \lim_{h \to 0} \frac{-h}{hx + h^2}$$

a) Identify the mathematical errors Joshua has made.

[2 marks]

b) Write a complete, rigorous proof for the derivation of the derivative of $y = \frac{1}{x}$ from first principles.

6 **a)** Find the first three terms, in ascending powers of x, of the binomial expansion of $\dfrac{1}{\sqrt[3]{8+2x}}$

[3 marks]

b) Hence, find the expansion of $\dfrac{1}{\sqrt[3]{8-2x^2}}$

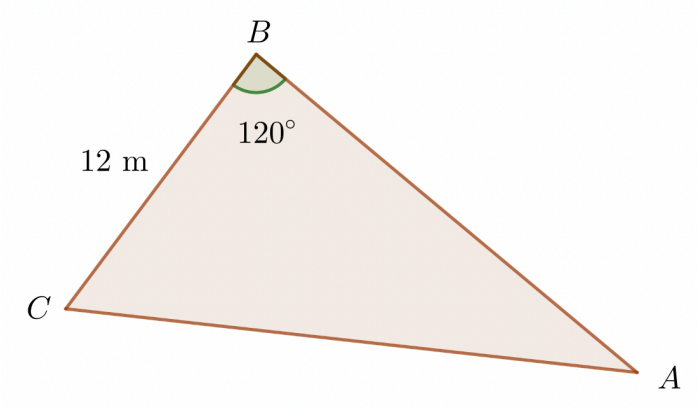
[2 marks]

c) Millie uses the first three terms of the expansion found in **(b)** to find an approximation to the the integral $\int_0^{\frac{1}{2}} \frac{2}{\sqrt[3]{8-2x^2}}\,\mathrm{d}x$.

Evaluate this approximation.

[3 marks]

7 Consider the triangle ABC of area 72 m².



a) Find the length of AB.

[2 marks]

b) Find the length AC.

c) Hence, find the smallest angle in the triangle.

[2 marks]

8 The differential equation below models the concentration of a reagent in a chemical reaction. When $t = 0$ seconds the scaled concentration takes a value of 2. Find the value of the scaled concentration when $t = 2$ seconds.

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{e}^{-y}(5t + 8)}{t^2 + 3t + 2}$$

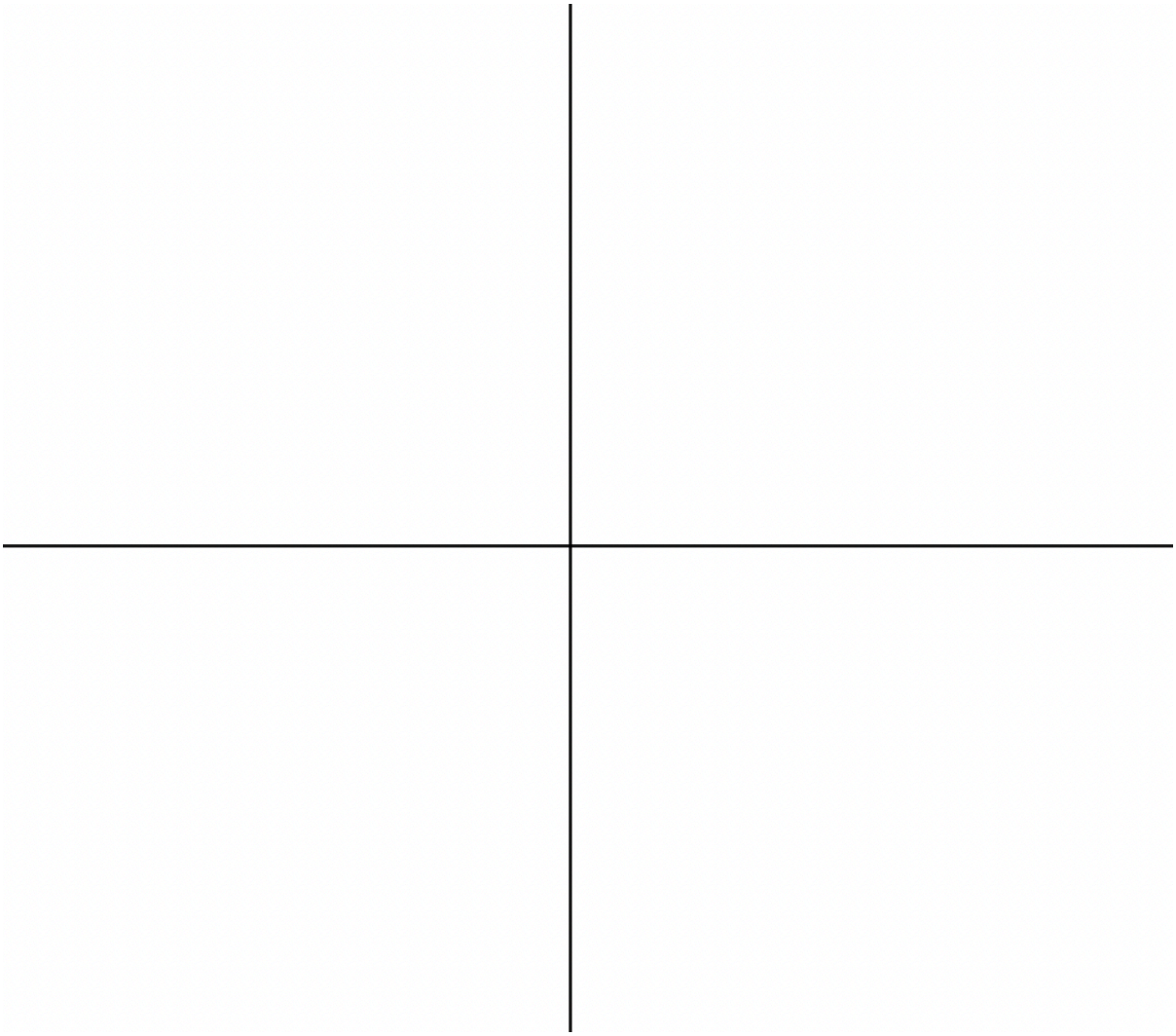
[8 marks]

9 **a)** Sketch the curve defined by the following

$$f(x) = \begin{cases} 2, & -2 \leq x \leq 1 \\ 2x, & 1 \leq x \leq 3 \\ -x + 9 & 3 \leq x \leq 5 \end{cases}$$

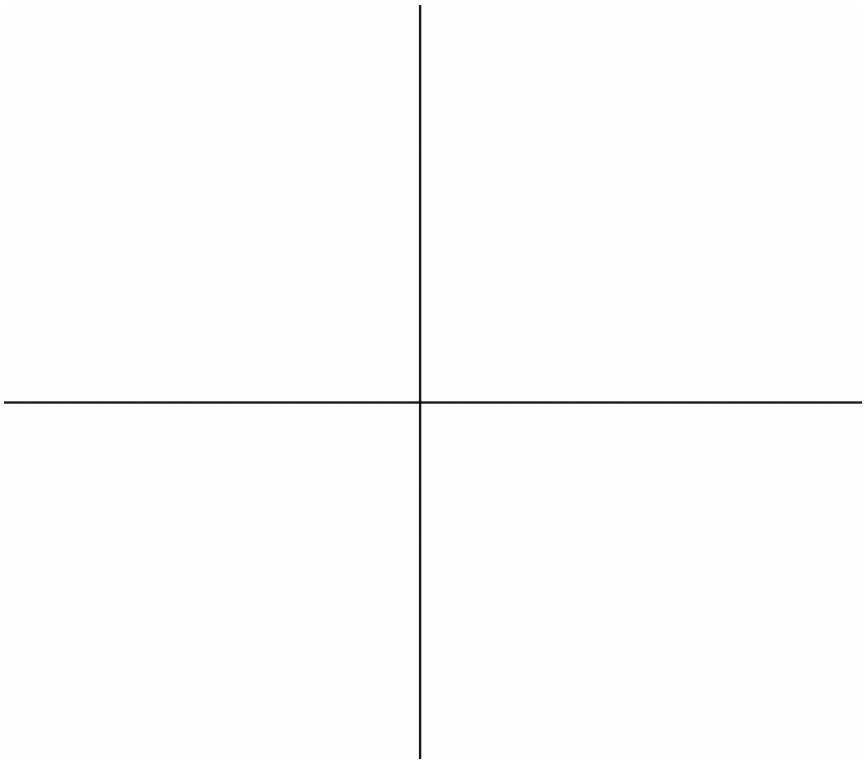
Label the points where $x = 1, x = 3$ and $x = 5$.

[2 marks]



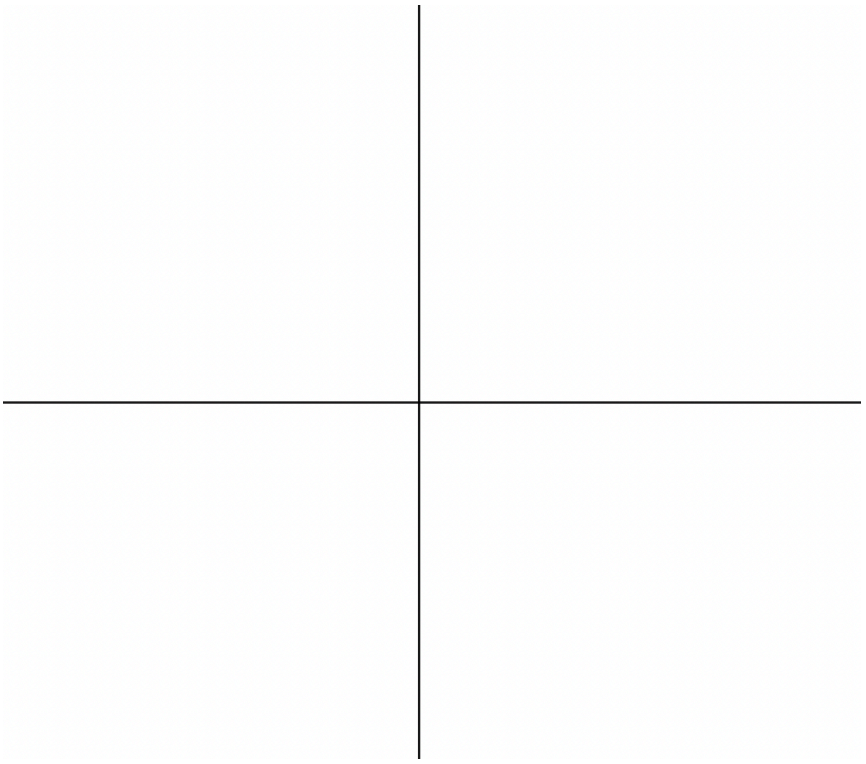
b) Hence, sketch $f(-x)$, showing the coordinates of the transformations of the points labelled in **(a).**

[2 marks]



c) Sketch $2f(x) - 4$, showing the coordinates of the transformations of the points labelled in **(a).**

[2 marks]



10 a) Find the points of inflection for the curve $y = \mathrm{e}^{-2x^2+x}$.

[5 marks]

b) Hence, find the regions where $f(x)$ is concave and convex.

[2 marks]

Section B

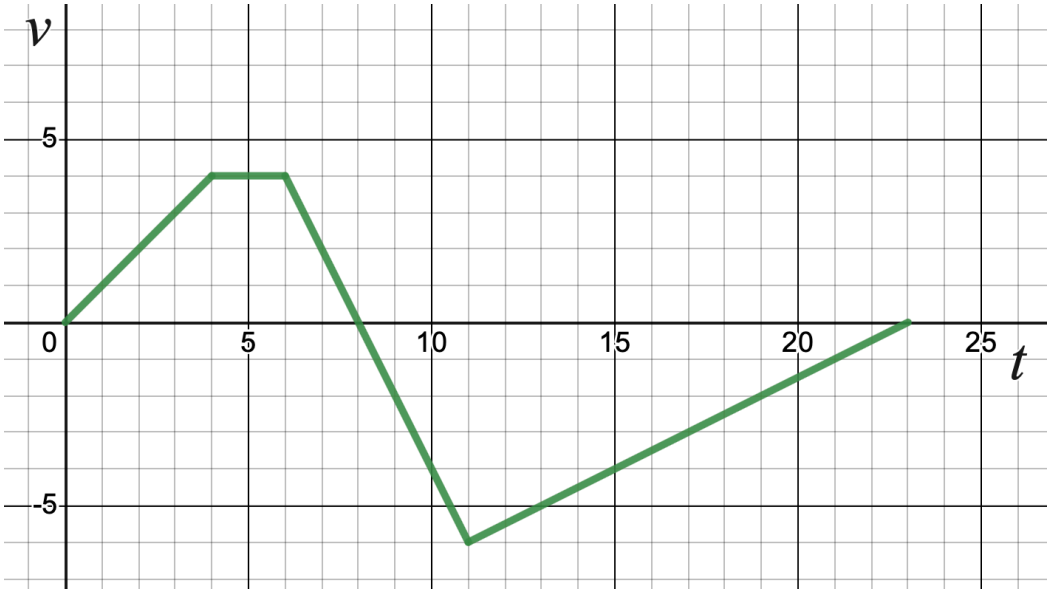
11 A number of forces act on a particle such that there resultant force is $\begin{pmatrix} -4 \\ 10 \end{pmatrix}$ N.

One of the forces is $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ N. Calculate the total of the other forces acting on the particle.

$\begin{pmatrix} -2 \\ -2 \end{pmatrix}$ $\quad$ $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$ $\quad$ $\begin{pmatrix} -6 \\ 8 \end{pmatrix}$ $\quad$ $\begin{pmatrix} -2 \\ 12 \end{pmatrix}$

[1 mark]

12 The velocity time graph below shows the velocity of a particle against time.



The total distance travelled is

0 m $\quad$ -25 m $\quad$ 65 m $\quad$ 25 m

[1 mark]

13 A particle of mass 3 kg moves in a horizontal plane under the action of a resultant force $\mathbf{F}$ newtons. The velocity of the particle is

$$\mathbf{v} = \begin{pmatrix} 18t + \cos(t) \\ \mathrm{e}^{-t} + \sin(t) \end{pmatrix}$$

a) Find an expression for the acceleration $\mathbf{a}$ as a function of t.

[2 marks]

b) **(i)** Find the magnitude of $\mathbf{F}$ at $t = 0$.

[2 marks]

(ii) In what direction is the force acting at this time?

[1 mark]

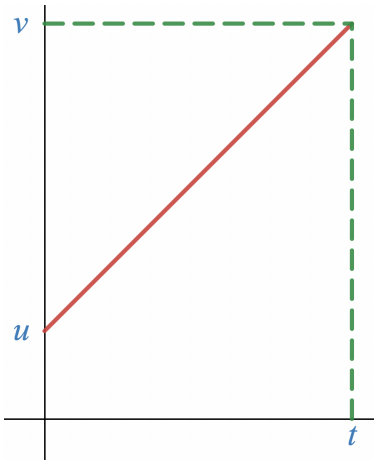
c) When $t = 0$ the particle is at the point with position vector $5\mathbf{i} + 2\mathbf{j}$.

Find the position vector $\mathbf{r}$ of the particle at time t.

[4 marks]

14

Using the graph shown below, or otherwise, show that $v^2 = u^2 + 2as$.



[4 marks]

16 A uniform plank, AB, of length 4 m and mass 20 kg is balanced on a pivot, P, which is 1 m away from A.

A mass of M kg is placed at A and a lady of mass 65 kg stands on the plank at B.

Given that the plank is in equilibrium, with neither A nor B touching the ground, find the value of M.

[3 marks]

17 A particle A moves on a horizontal surface with constant acceleration $-0.2\mathbf{i} + 0.1\mathbf{j}$ ms^{-2} and starts at the origin with velocity $3\mathbf{i} + 4\mathbf{j}$ ms-1.

a) Find the position vector, $\mathbf{r}$, of A t seconds after leaving the origin.

[2 marks]

b) Find the time it takes for the particle to reach a point due north of the origin.

[3 marks]

18 A block, of mass 2 kg is pulled across a rough horizontal surface by a force of 20 N inclined at an angle of 30° to the the horizontal. The coefficient of friction is 0.6.

a) Draw a force diagram of this situation.

[2 marks]

b) Find the acceleration of the block when it is under the action of this force.

[4 marks]

c) After 3 seconds of it being puled from rest the force is removed. How long until the block is again at rest?

[5 marks]

19 A ball is projected from the ground with speed u at an angle α to the horizontal where the only force acting on the ball is gravity.

The $x-$ and $y-$axes are horizontal and vertical, passing through the origin O in the plane of motion of the ball.

a) Find the time of flight of the particle.

[4 marks]

b) Show that the equation of the path can be given by

$$y = x\tan(\alpha) - \frac{gx^2}{2u^2}\left(1 + \tan^2(\alpha)\right)$$

[6 marks]

c) Find the range of the particle.

[3 marks]