AQA A-Level Maths 2022 Paper 1

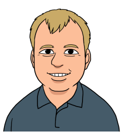
Do nut turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: / 100

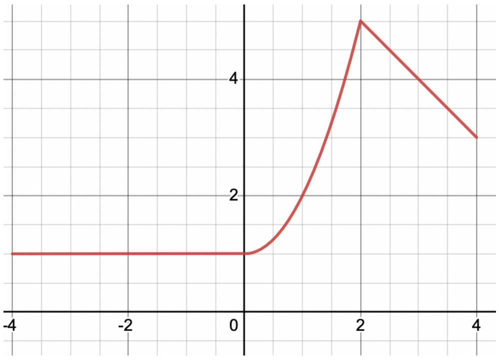


1 If $(2x+3)$ is a factor of $p(x) = ax^3 + bx^2 + cx + d$, then

$p\left(\frac{3}{2}\right) = 0$ $p\left(\frac{2}{3}\right) = 0$ $p\left(-\frac{3}{2}\right) = 0$ $p\left(-\frac{2}{3}\right) = 0$

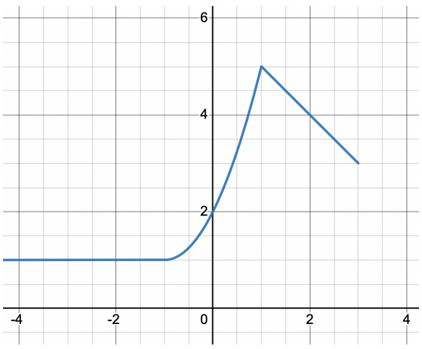
[1 mark]

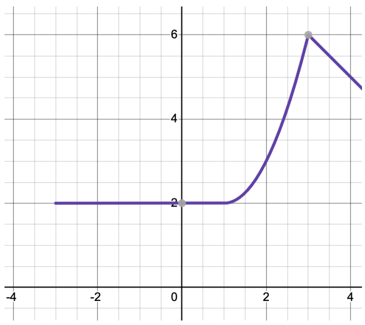
2 The function $f(x)$ is shown below

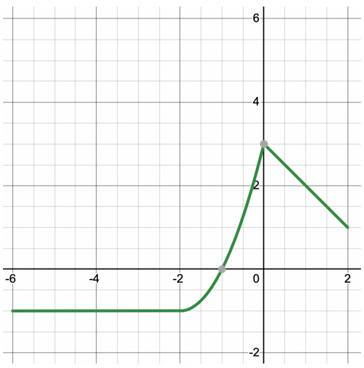


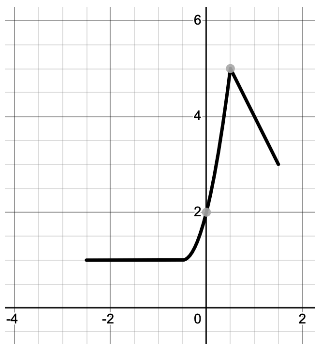
Which of the following graphs could not be a transformation of the form $f(x+a)+b$?

(1 mark)









3 **a)** Find the Cartesian equation of the curve with parametric equations $x = -3 + 2\cos(\theta), \quad y = 4 + 2\sin(\theta), 0 \leq \theta \leq 2\pi$.

[4 marks]

b) Sketch this curve.

[1 mark]

4 **a)** The line $y = mx + 15$ is tangent to the circle $(x-2)^2 + (y-1)^2 = 100$. Find the possible values of m.

[5 marks]

b) Find the coordinates of the point of intersection of the possible tangents.

[4 marks]

5 The fifth term of an arithmetic sequence is 22, and the tenth term of \ the same sequence is 37.

Find the maximum number of terms, n, such that the partial sum $S_n < 200$.

[7 marks]

6 The sequence generated by the recurrence relation $u_{k+1} = pu_k + q$,

$u_1 = 3$, where p is constant and $q = 6$ is periodic with order 2. Find the value of p.

[4 marks]

7 Fully justifying your answer, solve the equation,
$\cos^2(x) + \cos^3(x) + \cos^4(x) + \cdots = 1 + \cos(x)$ for $0 < x < 180$.

[10 marks]

8 With reference to graph transformations of the function $y = \sin(x)$ explain why the equation $\sin(2x) + \frac{1}{2} = 1$ has 4 solutions in the interval $0 \leq x \leq 2\pi$.

[2 marks]

9 Assuming θ is small enough so that terms involving θ^3 or higher may be ignored, find an expression in terms of θ for $\dfrac{\sin(2\theta)}{\cos(3\theta)} + \tan(\theta)$.

[4 marks]

10 Consider the curve given by

$$\sin(x) + \cos(y) = 1, \quad -\pi < x < \pi \text{ and } -2\pi < y < 2\pi.$$

Find the coordinates of the stationary points.

[8 marks]

11 **a)** Find where the tangent to the cubic $y = (x+1)(x-2)^2$ at $x = \dfrac{1}{2}$ meets the $x-$axis.

[6 marks]

b) What is significant about this value of x.

[2 marks]

c) Find the area bounded by this tangent and the $x-$ and $y-$ axes.

[4 marks]

12 a) The gradient function of a curve is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (1 + 2y)(3x + 1) .$$

Show that y can be written in the form $y = A\mathrm{e}^{3x^2+2x} - \dfrac{1}{2}$

[7 marks]

b) Given that $y(1) = 6$, find the value of the constant A.

[2 marks]

13 **a)** Show that $I = \int_1^2 \big((x+3)\ln(x)+2\big)\ \mathrm{d}x = \ln(256) - \dfrac{7}{4}$

[5 marks]

b) Find the percentage error in approximating I by using the Trapezium Rule with 5 ordinates.

[6 marks]

c) What would happen to this percentage error if the number of ordinates used in the Trapezium Rule was increased?

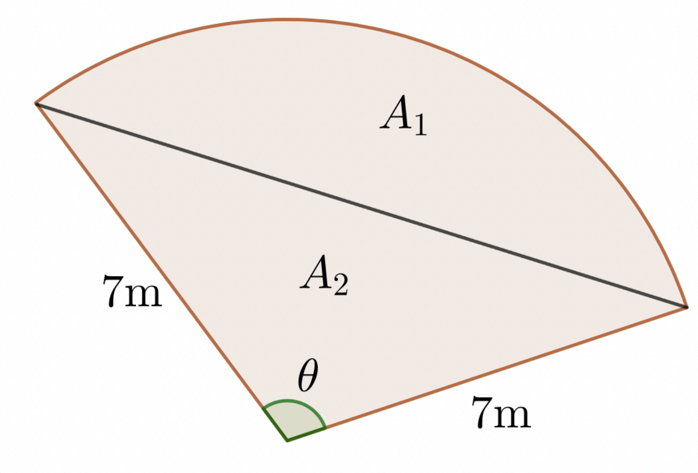
[1 mark]

14 Using the substitution $u = \tan(x)$ integrate

$$\int \sec^2(x) + 2\tan^2(x)\sec^2(x) + \tan^4(x)\sec^2(x)\ \mathrm{d}x$$

[6 marks]

15 The diagram below shows a large flowerbed that is in the shape of the sector of a circle. The gardener wishes to plant Tulips of one colour in the area A_1 and tulips of another colour in the area A_2. Given that he has the same number of bulbs of each tulip and he wants the planting density to be the same in A_1 and A_2.



a) Show that $98\sin(\theta) - 49\theta = 0$

[4 marks]

b) Show that a root of this equation lies in the interval $\left[1.5, 2\right]$.

[2 marks]

c) Find a better approximation to the root using the Newton-Raphson method twice, starting with $x_0 = 1.5$.

[4 marks]