Solutions / Answers

AQA AS-Level Maths 2022 Paper 1

Do nut turn over the page until instructed to do so.

This assessment is out of 100 marks and you will be given 120 minutes.

When you are asked to by your teacher write your **full name** below

Name:

Total Marks: **/ 80**

1 Find the area between the curve $y = \frac{1}{4}(x+4)(x-3)(x+2)$ and the $x-$axis.

$\frac{407}{16}$ $\quad$ $\frac{-343}{16}$ $\quad$ 2 $\quad$ $\frac{343}{16}$

[1 mark]

2 Which of the following is a counter example the statement "$x^3 + 3x - 5$ is always prime"

$x = 6$ $\quad$ $x = 5$ $\quad$ $x = 3$ $\quad$ $x = 4$

[1 mark]

3 Write as a single logarithm the following

$$10\log_2(x) + 2\log_2(yz^2) - 5\log_2(xz)$$

[3 marks]

$10\log_2(x) + 2\log_2(yz^2) - 5\log_2(xz)$

$= \log_2(x^{10}) + \log_2(y^2z^4) - 5\log_2(xz)$

$= \log_2\left(\frac{x^{10}y^2z^4}{x^5z^5}\right)$

$= \log_2\left(\frac{x^5y^2}{z}\right)$

4 **a)** Using the factor theorem, show that $(x + 4)$ is a factor of the polynomial $p(x) = 3x^3 + 14x^2 + 9x + 4$.

[2 marks]

$p(-4) = 0$

so by the factor theorem $(x+4)$ must be a factor of $p(x)$

b) Fully factorise $p(x) = 3x^3 + 14x^2 + 9x + 4$.

[1 mark]

$p(x) = (x+4)(3x^2+2x+1)$

c) Hence, prove that $p(x)$ has only one real root.

[3 marks]

Consider the discriminant of $3x^2+2x+1$

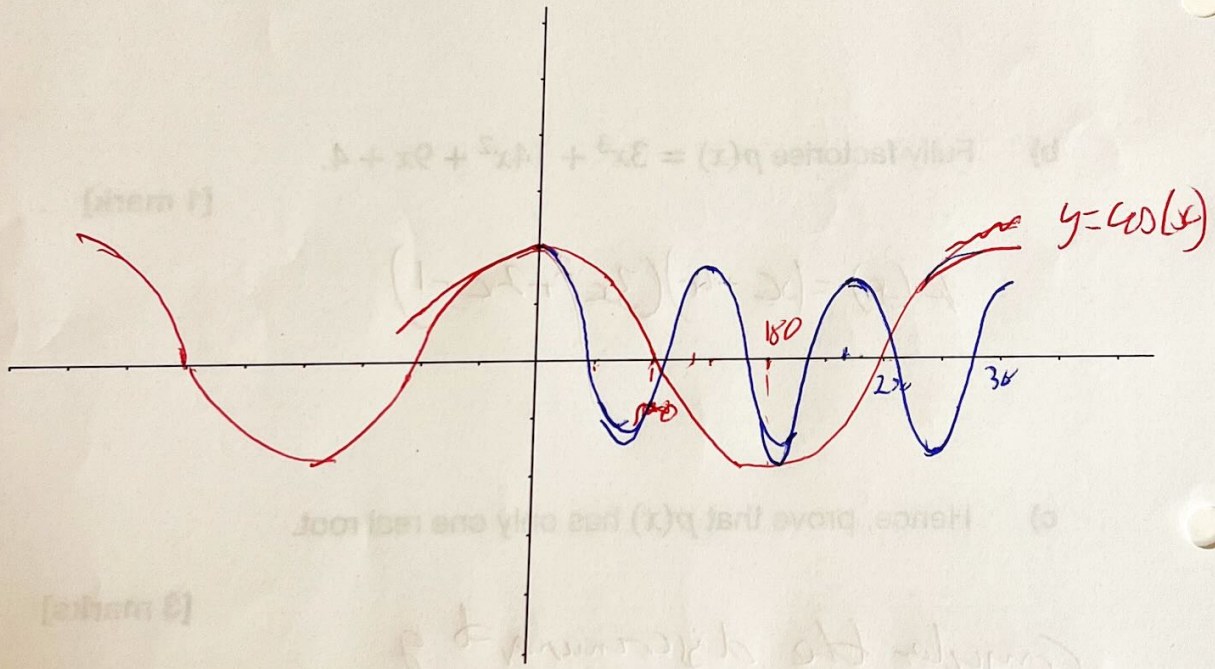
$\Delta = 2^2 - 4 \times 3 \times 1$

$= -8$

Since the discriminant < 0 there are no real roots of the quadratic $3x^2+2x+1$. Consequently, $p(x)$ has only one real root.

5 **a)** Sketch on the axes below $y = \cos(x)$ and $y = \cos(3x)$ for $-360^\circ \leq x \leq 360^0$

[3 marks]



b) How many solutions are there for the equation $\cos(3x) = \frac{1}{3}$ in the interval $0^\circ \leq x \leq 540^\circ$?

8

6 Consider the circle, C, given by

$$C : x^2 + y^2 + 6x + 4y - 12 = 0$$

a) Find the centre of C.

$(x+3)^2 + (y+2)^2 = 25$

centre $(-3, 2)$

[1 mark]

b) Find the radius of C.

radius 5

[1 mark]

c) The line $y = mx + 6$ is a tangent to the circle C. Find the value of m.

$m = -\frac{3}{4}$

[4 marks]

d) Hence, find the area bounded by the tangent and the $x-$ and $y-$ axes.

[3 marks]

Area = 24

7 **a)** Find the binomial expansion of $(2 + 3x)^4$

[4 marks]

$(2+3x)^4 = 16 + 96x + 216x^2 + 216x^3 + 81x^4$

b) The coefficient of the x^2 term in the expansion of $(2 + 3x)^4$ is the same as that of the x term in the expansion of $(2 + ax)^6$. Find a.

[2 marks]

$216 = \binom{6}{1} 2^5 a$

$\Rightarrow a = \frac{9}{8}$

8 Solve, in the interval $0° \leq x \leq 360°$,

$$10\sin^2(x) + 2\cos^2(x) + 10\sin(x) - 5 = 0$$

[6 marks]

$x = 14.47°$ and $x = 165.52°$

9 The curve $y = x^3 + px^2 - 5x + 7$ has a stationary point at $x = 5$.
Find the value of p and classify the stationary point.

[5 marks]

$p = 7$

minimum

10 Consider the function $f(x) = x^{\frac{3}{2}}\left(x^2 + 3\right) - 4x$.
Find the vertical distance between the points where the tangent and normal to $f(x)$ at $x = 1$ intersects the $y-$axis.

[10 marks]

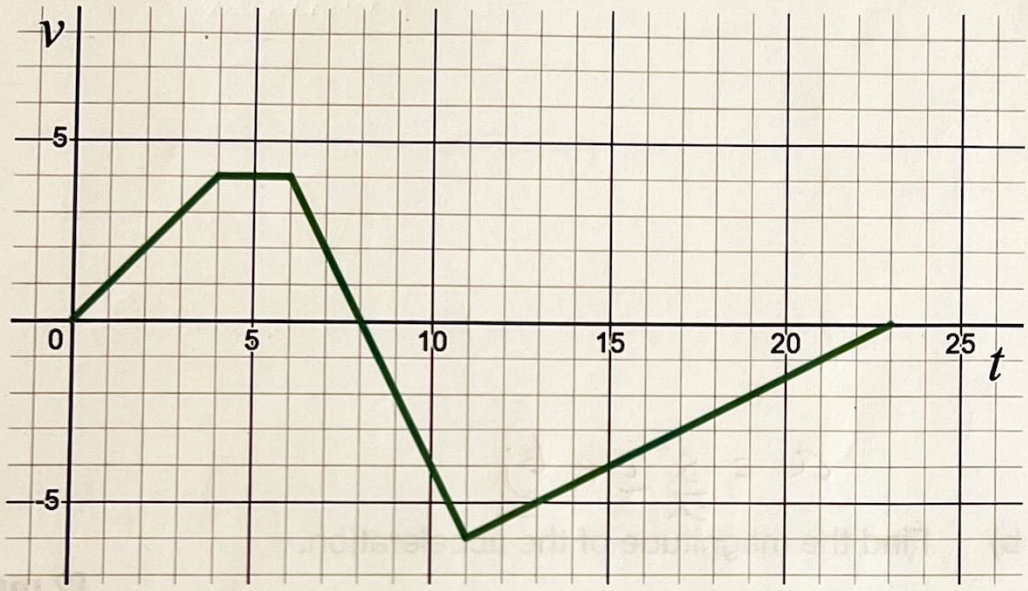
Tangent : $y = 4x - 4$, intersects y axis at $(0, -4)$

normal : $-x - 4y = -1$, intersects y axis at $(0, \frac{1}{4})$

So distance = 4.25

Section B

11 The velocity time graph below shows the velocity of a particle against time.



The total distance travelled is

0 m −25 m 65 m 25 m

12 A mass of 2 kg is acted on by a force $\mathbf{F} = 5\mathbf{i} + 12\mathbf{j}$.

a) Find the acceleration as a vector

[2 marks]

$a = \frac{5}{2}\underline{i} + 6\underline{j}$

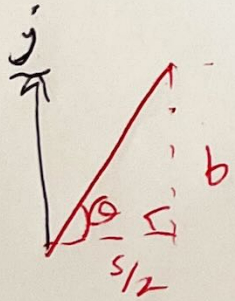
b) Find the magnitude of the acceleration.

[2 marks]

$\frac{13}{2}$

c) Find the direction of the acceleration relative to unit vector **j**.

[3 marks]



So $\theta = \arctan\left(\frac{6}{5/2}\right) = 67.38°$

So the acceleration acts at an angle of 22.61 degrees.

d) Compare the acceleration of this particle to one with acceleration $-10\mathbf{i} - 24\mathbf{j}$ ms^{-2}.

[1 mark]

$\begin{pmatrix} -10 \\ -24 \end{pmatrix} = -4\begin{pmatrix} 5/2 \\ 6 \end{pmatrix}$

So the accelerations are parallel, but in opposite directions

13 Three forces, $\mathbf{F}_1 = 2\mathbf{i} + 4\mathbf{j}$, $\mathbf{F}_2 = a\mathbf{i} + 7\mathbf{j}$ and $\mathbf{F}_3 = -8i + b\mathbf{j}$.

Given that resultant is $\mathbf{R} = 6\mathbf{i} + 5\mathbf{j}$, find a and b.

[3 marks]

$\begin{pmatrix} 2 \\ 4 \end{pmatrix} + \begin{pmatrix} a \\ 7 \end{pmatrix} + \begin{pmatrix} -8 \\ b \end{pmatrix} = \begin{pmatrix} 6 \\ 5 \end{pmatrix}$

$\Rightarrow \begin{cases} -6 + a = 6 \\ 11 + b = 5 \end{cases}$

$\Rightarrow a = 12$

and $b = -6$

14 A particle is projected vertically upwards with initial speed 10 ms^{-1}. Find how long the particle is above a height of 3 m.

For this question use the acceleration due to gravity to be 10 ms^{-2}.

↑ +ve

[3 marks]

s 3
u 10
v
a -10
t

Using $s = ut + \frac{1}{2}at^2$, $\Rightarrow 3 = 10t - 5t^2$

$\Rightarrow 5t^2 - 10t + 3 = 0$

$\Rightarrow t = 1.63$, and 0.37,

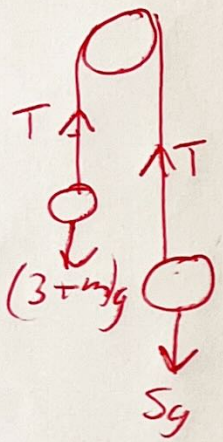
Hence the particle is above 3m for
$1.63 - 0.37 = 1.26$ seconds.

15 A particle of mass $(3 + m)$ kg is connected by a light inextensible string to another particle of mass 5 kg.

The string is then placed over a smooth pulley and the system is held at rest.

a) Show that, when released, the acceleration felt by the particles has magnitude $a = \dfrac{(2+m)g}{8+m}$.

[3 marks]



Apply F=ma ↑ for A $T - (3+m)g = (3+m)a$ (1)

Apply F=ma ↓ for B $5g - T = 5a$ (2)

(1) + (2)

$5g - (3+m)g = (3+m)a + 5a$

$g(2+m) = a(8+m)$

$\Rightarrow a = \dfrac{(2+m)g}{8+m}$

b) How have you used the fact that the pulley is smooth in your working above?

[1 mark]

The tensions either side of the pulley will be the same

16 A particle is moving in a horizontal straight line from A to B. It takes 6 seconds to move the 30 m from A to B. At B the velocity of the particle is 2 ms^{-1}.

a) Find the velocity of the particle at A.

[3 marks]

$s = 30$

$u =$

$v = 2$

$a =$

$t = 6$

$$30 = \left(\frac{u+2}{2}\right)6$$

$$\Rightarrow 10 = u + 2$$

$$\Rightarrow u = 8 \text{ ms}^{-1}$$

b) Find the acceleration experienced by the particles as it moves from A to B.

[3 marks]

$s = 30$

$u =$

$v = 2$

$a =$

$t = 6$

$s = vt - \frac{1}{2}at^2$

$30 = 2 \times 6 - \frac{1}{2} \times a \times 6^2$

$30 = 12 - 18a$

$\Rightarrow 18 = -18a$

$\Rightarrow a = -1 \text{ ms}^{-2}$

17 A block of mass 3 kg is moving with velocity $v(t) = 2e^{3t} + t$.

a) Find the acceleration when $t = 2$ s.

[2 marks]

$a = \frac{dv}{dt}$

$= 6e^{3t} + 1$

When $t = 2$

$a = 6e^6 + 1 \text{ ms}^{-2}$

b) What is the force experienced by the particle at this instant?

[2 marks]

$F = ma$

$= 3 \times (6e^6 + 1)$

$= 18e^6 + 3 \text{ N}$